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Abstract

Recently, many modern industries have adopted joint scheduling of production and distribution decisions. Such coordination is necessary in make-to-order (MTO) businesses, where it is challenging to achieve timely delivery at minimum total cost and meet the requirements for high customization. To deal with these challenges, a practical production configuration and delivery method is required, in addition to a closer link between production and distribution schedules. Hence, in this study, we address an integrated production scheduling-vehicle routing problem with a time window, where it is assumed that production is performed in a flexible job-shop system. Our framework is modeled as a novel bi-objective mixed integer problem, in which the first objective function aims to minimize a sum of the production and distribution scheduling costs, and the second objective function tries to minimize a weighted sum of delivery earliness and tardiness. To practically validate the application of our framework, a case study from a furniture manufacturing company producing customized goods is considered, and experimental data are derived. Based on the real data, the model is first optimally solved by an $\varepsilon -$constraint method, and then a Hybrid Particle Swarm Optimization (HPSO) algorithm is developed to solve the model for medium- and large-sized problems in a reasonable time. We discuss the benefits of integration by comparing the results of the proposed model with that of the separate approach. The results show that the company can establish a proper rational balance between cost and customer concerns, and they can use the integration policy as a lever to improve customer satisfaction without the system experiencing a significant increase in total operational cost.

Keywords: Supply chain scheduling, multi-objective optimization, integrated production-distribution, $\varepsilon -$constraint method, hybrid particle swarm optimization algorithm.

1. Introduction

Over the past decade, in real-world production industries, growing attention has been devoted to make-to-order (MTO) business, partially owing to their benefits in reducing finished products' inventory level, high customization, and adapting to rapidly-changing customer behavior. In such environments, finished products must be delivered to customers shortly after their production. More specifically, a company in an MTO environment must not only manage producing a high variety of customized products, they must also provide an on-time delivery. Nevertheless, timely delivery is pointless if it results in higher costs owing to excessive use of production and distribution resources. Therefore, to suitably optimize an MTO business, it is important to develop a level-headed optimization scheme that can simultaneously manage scheduling of production and distribution decisions for custom-made products.

Traditional optimization of production and distribution schedules is performed separately and sequentially, such that jobs are first processed in a production facility without considering distribution decisions, and then finished products are delivered. Indeed, the outputs of the production scheduling become the inputs of the delivery scheduling. It is recognized that traditional approach, based on merely minimizing cost, not only fails to reduce the whole cost of the supply chain, but also fails to satisfy customers' expectations for timely delivery.

Thus, it is important to design a professional scheduling plan along with a suitable production and delivery configuration to adequately cope with MTO business optimization challenges, including high customization in customer quality and service standards, costly logistics services, and price and delivery time competition in the market. Thus, this study addresses an important variation of an integrated production-distribution scheduling (IPDS) problem, where production is performed in a *Flexible Job-Shop* (FJS) system, allowing us to have more flexible routing of the process, and consequently a more impressive effect on the distribution side of the system. Despite FJS becoming an inevitable part of modern innovative industries, its optimization models and respective solving procedures are more complicated, and thus these types of systems are rarely discussed in the IPDS models' literature. In that regard, a timely delivery of finished products, as performed by vehicle routing as a cost-effective delivery method with time windows and heterogeneous vehicles, is addressed by a limited number of IPDS studies.

Our research efforts concern not only the economic aspects of the integrated FJS scheduling-vehicle routing decisions, but also the minimization of violations to the imposed delivery time windows. More importantly, we aim to find a joint scheduling optimization scheme for order processing in the production facility and order delivery, to simultaneously achieve the minimal cost of production and distribution scheduling and the highest possible level of customer satisfaction. We specifically study a situation where an integration strategy can play a key role in not only fulfilling customers' expectations, but where, as we will illustrate, it is also capable of keeping production and distribution operations as economical as possible. Additionally, our bi-objective structure allows us to derive a trade-off between cost and a minimal weighted sum of delivery earliness and tardiness, by using the integration option. In addition, solving a proposed model for a case study from a real MTO business enables us to practically demonstrate the applicability of our framework.

The remainder of this paper is organized as follows: the next section reviews related works on the IPDS problem to derive the research gap and emphasizes the main contributions of this paper. Section 3 provides the problem description, as well as the case study which inspired us to develop it. The problem description is then followed by the mathematical formulation of the proposed model. Section 4 details the resolution techniques, namely the $\varepsilon-$constraint, and the developed hybrid particle swarm optimization (PSO) algorithms. Section 5 proves the applicability of our model on a small-sized example and on several medium- and large-sized test problems. We particularly show how our proposal permits decision makers to judge and tradeoff economic considerations and the customer time window satisfaction. Section 6 deals with an additional managerial insight pertaining to our model. We particularly show the benefits resulting from the integration of both production and distribution decisions. The conclusion and directions for future research are presented in Section 7.

2. Literature review

Recently, growing attention has been devoted to the IPDS problem, which can be generally classified according to the machine configuration, e.g., multi-purpose or single task machines, delivery method, e.g., homogenous or heterogeneous types of vehicles, single or multi-objective frameworks of models, and the chosen objectives of the problem. Here, we review the most-related IPDS literature regarding a single factory and outbound delivery. Below, all variants of machine configurations and delivery methods are respectively represented.

- Machine configurations: single machine, parallel machines, flow-shop configuration, open-shop configuration, job-shop configuration, bundling operation.
- Delivery methods: Single delivery, direct delivery, routing, split delivery.

An overview of related IPDS studies based on the above classification is provided in Table 1. Despite the fact that the integrated production and distribution problem at the scheduling level has received significant attention in recent years, the majority of the publications have restricted their focus to simple machine configurations, including: single machine (e.g. hall and Potts, 2003; Low et al., 2014; Li et al., 2016), identical parallel machines (e.g. Garcia & Lozano, 2005; Ullrich, 2013; Liu and Lu, 2016), and unrelated parallel machines (Chang et al., 2013, Guo et al., 2015, Joo and Kim, 2016). More specifically, very few studies on IPDS (Soukhal et al., 2005; Li and Vairaktarakis, 2007; Hassanzadeh et al., 2016) looked at the problem of order processing in multi-stage production systems, such as flow-shop, job-shop, open-shop, and bundling operation environments, where the completion of a job follows the processing of a given set of operations (process routing). Although these researchers addressed an IPDS problem by considering process routing, they assumed only a flow-shop or bundling manufacturing system, for special cases of the problem. Moreover, their studies involved some unrealistic and simple assumptions, and never investigated practical features such as the general size of orders, heterogeneous vehicles, and delivery due dates (time windows). In addition, these works studied either direct shipment (Soukhal et al., 2005) or batch selection as a simple delivery methods (Hassanzadeh et al., 2016), and no routing delivery is involved in these studies, except in the work by Li and Vairaktarakis (2007), which addressed a milk run delivery as a special case of a vehicle routing problem (VRP). Moreover, according to Table 1, very few investigations on IPDS proposed a multi-objective model to elaborate on the inherent conflicts between cost and customer concerns (Cakici et al., 2012; Li et al. 2016; Hassanzadeh et al., 2016). Furthermore, the existing bi-objective models did not consider production cost in the objective functions and did not study the impact of the integration on these two conflictive criteria, to validate the applicability of their proposed integrated models. Consequently, the question of how integration can act the role of a lever in a bi-objective framework to economically boost customer satisfaction has never been investigated by IPDS professionals. We also note that the IPDS literature has not yet reported on complicated manufacturing systems with a high range of customized products and flexible process-oriented systems. The reader is referred to Chen (2010) and Moons et al. (2016) for a comprehensive review on IPDS.

Our study contributes to the existing literature by investigating the following four issues. First, we examine a comprehensive real-world inspired scheduling problem, in which optimization of an FJS system in the presence of multi-purpose machines is integrated with a vehicle routing problem under time window (VRPTW) constraints, in an MTO supply chain. It is known that job-shop is a type of manufacturing process that fits the production of a high variety of customized products. However, as shown in Table 1 and to the best of our knowledge, none of the IPDS investigations concerning MTO business (e.g. Chang et al., 2013) studied the job-shop scheduling problem. Moreover, to the best of our knowledge, the current study is among the first investigations studying a flexible machine scheduling problem by considering transportation decisions. Second, in addition to filling the aforementioned research gaps, our study concerns a practical routing delivery problem with a *soft time-window*, meaning that a violation to the time windows is allowed, which provides a degree of flexibility in routing but can also degrade customer satisfaction. This is in contrast to hard time windows, where a violation to the time window is forbidden. Additionally, we consider different sizes of orders and a heterogeneous fleet composed of vehicles with different capacities, and fixed and variable costs.

Third, we combine and trade-off two conflicting performance measures in a bi-objective modeling framework: cost minimization, and customer satisfaction maximization. The cost function models joint production scheduling and distribution expenditures; whereas, the customer satisfaction models the time window aspects. The distribution cost depends on the number of vehicles (fixed cost) and the total distance traveled by each vehicle (variable cost), and the production cost includes the cost of operations processed on each machine. Fourth, and from a theoretical point of view, we model our problem as a generic bi-objective model. Thus, our model could be adapted to solve other production and/or distribution systems with simpler machine (such as flow-shop) and/or delivery configurations (such as direct shipment and split delivery).

Table 1. *Overview of the related integrated production-distribution scheduling (IPDS) studies*

Author(s)	Flexible job-shop	VRP	Process routing	Multi-purpose machines	Heterogeneous vehicles	Objective (s)	Multi objective	Solution procedure	Other features
Hall & Potts 2003	-	-	-	-	-	DC + CS	-	Dynamic programming	Different scenarios
Chen & Vairaktarakis 2005	-	✓	-	-	-	DC + CS	-	Heuristic, dynamic programming	Different scenarios, make-to-order (MTO)
Garcia & Lozano 2005	-	-	-	-	-	Max Revenue	-	Tabu search, exact	Time window
Pundoor & Chen 2005	-	-	-	-	-	DC + CS	-	Heuristic, dynamic programming	MTO supply chain
Soukhal et al. 2005	-	-	✓	-		Min CT	-	-	-
Li & Vairaktarakis 2007	-	milk run	✓	-	-	DC + CS	-	Heuristic	-
Stecke & Zhao 2007	-	-	-	-	✓	Min DC	-	Heuristic	MTO supply chain
Amstrong et al. 2008	-	✓	-	-	-	Max CS	-	Heuristic, B&B	MTO supply chain, time window
Geismar 2008	-	✓	-	-	-	Max CS	-	Heuristic	Short life span products
Chen & Pundoor 2009	-	-	-	-	-	Min DC	-	Heuristic	MTO
Cakici et al. 2012	-	-	-	-	✓	Min DC Max CS	✓	Heuristic	-
Viergutz & Knust 2012	-	✓	-	-		Max Service level	-	Heuristic	MTO, Time window
Chang et al. 2013	-	✓	-	-	-	DC + CS	-	Ant colony algorithm	MTO
Ullrich 2013*	-	✓	-	-	✓	CS	-	Decomposition, genetic	Time window
Low et al. 2014	-	✓	-	-	✓	DC + CS	-	Genetic algorithm (GA),	Time window
Guo et al. 2015	-	-	-	-		PC+DC + CS	-	Harmony search based memetic	MTO, multiple transportation modes
Devapriya et al. 2016	-	✓	-	-	-	DC + CS	-	GA, memetic algorithm	Time sensitive products
Hassanzadeh et al. 2016	-	-	✓	-		Min DC Max CS	✓	MOPSO, NSGA-II	-
Joo & Kim 2016	-	-	-	-	✓	Min CT	-	Genetic algorithm	-
Li et al. 2016	-	✓	-	-	-	Min DC	✓	NSGA-II	-

						Max CS			
Liu and Lu 2016	-	-	-	-		Max CS	-	Approximation algorithm	-
Karaoglan & Kesen 2016	-	✓	-	-	-	Max CS	-	B&B, simulated annealing	Time sensitive product
Saglam & Banerjee 2017	-	-	-	-	-	Min PC+DC + holding cost	-	Exact	-
Our study	✓	✓	✓	✓	✓	Min PC+DC Max CS	✓	Hybrid PSO, ε-constraint	MTO, time window

PC: Production cost, DC: Distribution cost, CS: Customer satisfaction, CT: Completion time,
* Ullrich (2013) assumed that vehicles are heterogeneous which differ in fixed cost and ready time

3. Problem statement

This section first describes the investigated IPDS problem, then introduces the notations to be used in this study, and finally presents our proposed mathematical model.

3.1. Problem definition

We consider an MTO manufacturing system that produces a wide variety of furniture for different customers. At the beginning of the planning horizon, each customer places exactly one order, with a specified size and a time window within which the order should be delivered. The manufacturing company is responsible for producing received orders and transporting finished goods to the corresponding customers. As the products are custom-made, they must be delivered shortly after they are produced, in their imposed time windows. However, timely delivery requires more resources, such as a larger number of shipments in a delivery, that will increase the manufacturer's total cost. Therefore, the problem faced by the company is to jointly optimize production and distribution scheduling decisions, by minimizing the related total scheduling cost while meeting the committed time windows for order fulfillments.

Production is performed in an FJS manufacturing system, with a set of multi-purpose machines. Each job comprises a fixed set of operations, each of which can be processed by different machine types in different processing times. However, each operation should be processed by one of the candidate machines, and each machine can process only one operation at a time. Therefore, in the production side, the company needs to determine which operation should be assigned to which machine through a given set of machines, and how to schedule assigned operations on each machine. Moreover, transportation is performed with a limited number of heterogeneous vehicles having different capacities, and fixed and variable costs. After jobs are completed, they are batched together in different vehicles for delivery to respective customers. The total size of each delivery batch must not exceed the capacity of the vehicle. Each vehicle is initially stationed in the production facility, and after serving the specified route, returns to the production facility. It is realistically assumed that the starting time of each vehicle's tour is equal to the completion time of the last job in the vehicle's cargo. Hence, on the distribution side, the company should determine how many shipments of each vehicle type to use, which customer should be served in which trip, the sequence of orders in each trip, and schedule the time for the departure of each shipment from the production site. We model the problem as a joint production-distribution scheduling framework and propose a novel bi-objective mixed integer model, trying to identify the best path and schedule of operations through machines, and the optimal arrangement of vehicles, routes, and the ideal departure time of vehicles from the production site. This is to simultaneously satisfy the two conflicting objectives: minimizing the production and distribution scheduling costs and minimizing the weighted sum of delivery earliness and tardiness. In the proposed model, the distribution cost involves a fixed charge for each of the vehicles used, and a variable cost in the total distance of the route taken by the vehicles. In a job-shop environment, a job may be processed by different machine types and probably by different processing times and costs, and it is therefore necessary to also consider the production cost as an important component of the total cost of system.

The framework described earlier is inspired from a real case study, from which we extract and use data in our numerical experiment. The company operates in the wood furniture sector, and they are subject to increasing pressure from product customization and growth in customer concerns. Adopting the MTO business model and producing custom-made products are becoming more and more relevant in this industry sector. One of the most famous and well-established customized-furniture companies (established in 1995 in the Middle East) is used as a case study herein. When it was created, the company started its activity by producing folding beds. Today, it produces a wide range of space-saving modular furniture. In 2001, with progress in technology based on the presence of modern machines, the company mechanized its production system, and improved customer satisfaction through developing the diversity and quality of the produced goods. In recent years, the company has expanded the range of its product catalog, and they have given customers a higher degree of product customization. Figure 1 illustrates some of the products offered by the company (office and other types of furniture, tables and chairs, drawers, bedding set, child bedding set, folding sets such as bed and furniture), and clearly shows the diversity and customization challenges facing the company.

Figure 1. *Various types of customized goods produced by the company* under studied

3.2. Notations

In this section, the notations used in discussion of the problem under investigation are introduced as follows:

Sets

$\omega = \{1, \dots, N\}$	set of parts, indexed by i, j,
$\eta_j = \{1,2, \dots, R_j\}$	set of operations of part j, indexed by f, r
$\Phi = \{1,2, \dots, M\}$	set of machines, indexed by m
$\Omega = \{1,2, \dots, V\}$	set of available vehicles, indexed by v

Parameters

λ_m	unit processing cost on machine m per unit of time
F_v	fixed cost of vehicle type v

θ_v	variable cost of vehicle type v per unit of time
μ	weight of early delivery
φ	weight of tardy delivery
P_{jrm}	processing time of operation r of job (order) j on machine m
a_{jrm}	process-machine matrix, which takes value 1 if machine m enables to process operation r of job j; otherwise it is equal to 0
t_{ijv}	transportation time from customer i to customer j by vehicle v
δ_j	size of job j
(a_j, b_j)	delivery time window of job j
Q_v	capacity of vehicle v

Decision variables

-Production side

π_{jr}	production start time of operation r of job j
Υ_{jr}	production completion time of operation r of job j
C_j	production completion time of job j
X_{jrm}	a binary variable which takes the value 1 if operation r of job j is processed by machine m, and 0; otherwise.
Y_{ifjrm}	a binary variable which takes the value 1 if operation r of job j is processed immediately after the operation f of job i, both on machine m, and 0; otherwise.

-Distribution side

D_j	delivery time of order j
T_{jv}	visiting time of customer (order) j by vehicle v
S_v	leaving time of vehicle v from production facility
E_v	visiting time of the last customer (order) in the tour of vehicle v
Z_{jv}	a binary variable which takes the value 1 if job j is delivered by vehicle v, and 0; otherwise.
U_{ijv}	a binary variable which takes the value 1 if job j is delivered after job i, by vehicle v, and 0; otherwise.
W_v	a binary variable which takes the value 1 if vehicle v is used for delivery, and 0; otherwise.

3.3. Mathematical model

The investigated IPDS problem is formulated as the following mixed integer nonlinear model:

$$\min f_1 = \underbrace{\sum_{j=1}^{N}\sum_{r=1}^{R_j}\sum_{m=1}^{M} \lambda_m P_{jrm} X_{jrm}}_{f11} + \underbrace{\sum_{v=1}^{V}[F_v W_v + \theta_v (E_v - S_v)]}_{f12} \quad (1)$$

$$min\ f_2 = \varphi \times \sum_{j=1}^{N} max(D_j - b_j, 0) + \mu \times \sum_{j=1}^{N} max(a_j - D_j, 0) \quad (2)$$

Subject to:

$$\sum_{m=1}^{M} X_{jrm} = 1 \qquad \forall j \in \omega, \ \forall r \in \eta j \qquad (3)$$

$$X_{jrm} \leq a_{jrm} \qquad \forall j \in \omega, \forall r \in \eta j, \forall m \in \Phi \qquad (4)$$

$$X_{ifm} = \sum_{j=1}^{N+1} \sum_{r=1}^{R_j} Y_{ifjrm} \qquad \forall i \in \omega, \ \forall f \in \eta j, \forall m \in \Phi \qquad (5)$$

$$X_{jrm} = \sum_{i=0}^{N} \sum_{f=1}^{R_i} Y_{ifjrm} \qquad \forall j \in \omega, \ \forall r \in \eta j \ \ \forall m \in \Phi \qquad (6)$$

$$\pi_{jr} \geq max\left\{ Y_{j(r-1)}, \sum_{i=0}^{N} \sum_{f=1}^{R_i} \sum_{m=1}^{M} Y_{if} \times Y_{ifjrm} \right\} \qquad \forall j \in \omega, \ \forall r \in \eta j \qquad (7)$$

$$Y_{jr} = \pi_{jr} + \sum_{m=1}^{M} P_{jrm} \times X_{jrm} \qquad \forall j \in \omega, \ \forall r \in \eta j \qquad (8)$$

$$C_j = Y_{jR_j} \qquad \forall j \in \omega \qquad (9)$$

$$\sum_{j=1}^{N} \sum_{f=1}^{R_0} \sum_{r=1}^{R_j} Y_{[0]fjrm} = 1 \qquad \forall m \in \Phi \qquad (10)$$

$$\sum_{i=1}^{n} \sum_{f=1}^{R_i} \sum_{r=1}^{R_{N+1}} Y_{if[N+1]rm} = 1 \qquad \forall m \in \Phi \qquad (11)$$

$$\sum_{v=1}^{V} Z_{jv} = 1 \qquad \forall j \in \Omega \qquad (12)$$

$$Z_{jv} = \sum_{i=0}^{N} U_{ijv} \qquad \forall j \in \omega, \forall v \in \Omega \qquad (13)$$

$$\sum_{i=0}^{n} U_{ijv} = \sum_{i=1}^{N+1} U_{jiv} \leq 1 \qquad \forall j \in \omega, \forall v \in \Omega \qquad (14)$$

$$\sum_{j=1}^{N} U_{[0]jv} = \sum_{i=1}^{N} U_{i[N+1]v} \leq 1 \qquad \forall v \in \Omega \qquad (15)$$

$$\sum_{j=1}^{N} Z_{jv} \delta_j \leq Q_v \qquad \forall v \in \Omega \qquad (16)$$

$$T_{[0]v} = S_v = \max_{j \in \omega} Z_{jv} C_j \qquad \forall v \in \Omega \qquad (17)$$

$$T_{jv} = \sum_{i=0}^{N} U_{ijv}\left(T_{iv} + t_{ijv}\right) \qquad \forall j \in \omega, \forall v \in \Omega \tag{18}$$

$$D_j = \sum_{v=1}^{V} Z_{jv} T_{jv} \qquad \forall j \in \omega \tag{19}$$

$$E_v = S_v + \sum_{i=0}^{N} \sum_{j=1}^{N+1} U_{ijv} t_{ijv} \qquad \forall v \in \Omega \tag{20}$$

$$W_v = \max_{j \in \omega} Z_{jv} \qquad \forall v \in \Omega \tag{21}$$

$$X_{jrm}, Y_{ifjrm}, Z_{jv}, U_{ijv}, W_v \in \{0,1\} \qquad \forall i,j \in \omega, \forall f,r \in \eta j, \forall v \in \Omega, \forall m \in \Phi \tag{22}$$

$$\pi_{jr}, \Upsilon_{jr}, C_j, D_j, T_{jv}, S_v, E_v \geq 0 \qquad \forall j \in \omega, \forall r \in \eta j, \forall v \in \Omega \tag{23}$$

Equation (1) is the first objective function, and represents the total cost of the production-distribution system, including machine processing cost (f_{11}) and distribution cost (f_{12}). The latter is composed of a fixed cost function of the vehicles utilized, plus a variable cost function of the total distance of the routes taken by the vehicles. Equation (2) introduces customer satisfaction, and it aims to minimize the weighted sum of delivery earliness and tardiness. Constraints (3) guarantee that each operation of each job must only be assigned to one machine. Constraints (4) denote that each operation of each job should be assigned to a machine that is able to process it. Constraints (5) and (6) restrict operations such that each operation on each machine has only one operation before it, and only one other operation after it, respectively. Orders *0* and *N+1* are two dummy orders having processing time 0 ($P_{[0]rm} = 0$). At the beginning of each machine processing, operation *r* of order *0* must be processed first, and operation *r* of order *N+1* must be processed last. In addition, the completion time and also starting time of operation *r* belonging to order *0* and *N+1*, are 0 ($\Upsilon_{[0]r} = 0$, $\pi_{[0]r} = 0$, $\Upsilon_{[N+1]r} = 0$, $\pi_{[N+1]r} = 0$). Constraints (7) guarantee that each operation of each job can at least begin when the completion of its predecessor operation ($\Upsilon_{j[r-1]}$) on any machine is finished, and the respective machine is not busy.

Constraints (8) calculate the completion time of each operation, which is equal to its starting time plus its processing time on the respective machine. Constraints (9) calculate the completion time of each job j. Constraints (10) ensure that only one operation is processed first on each machine m. Constraints (11) guarantee that only one operation is processed last on each machine m. Constraints (12) denote that each job should be only assigned to one of the available vehicles. Constraints (13) restrict each order in each tour to having only one order before it. Constraints (14) specify that each vehicle should leave immediately after delivery of the assigned orders to the related customers. Constraints (15) guarantee that each vehicle begins its tour from the production plant and returns to it only once. Here, in the distribution scheduling, two dummy orders are used to show that in each delivery batch, order 0 first departs from the company, and order $n+1$ at the end of each tour returns to it. Constraints (16) guarantee that the capacity of each vehicle is not exceeded by the total size of the orders. Constraints (17) specify that the departure time of each vehicle is equal to the biggest production completion time of all jobs in the batch. Constraints (18) denote that the delivery time of order j in vehicle v is equal to the receiving time of the prior order i by this vehicle, plus the traveling time between customers i and j. Equations (19) give the calculation of the delivery time of each order in each trip. Constraints (20) imply that the end time of each tour is equal to its starting time from the production site plus the total time of the routes taken by each vehicle. Constraints (21) determine which vehicles are used for delivery. Finally, Constraints (22) and (23) define the variable types.

3.4. Linearization

The proposed model is a nonlinear mixed-integer programming model. Before solving the model, we use some theoretical techniques to make the model linear, and consequently more tractable.

As seen in Equation (2) and Constraints (7) and (21), we used the *maximum operator*, which is an explicit nonlinear term. The *max* (*min)* operator is linearized by default when *IBM ILOG CPLEX* is used, by the help of the *maxl (minl)* function. Theoretically, to make the proposed model more efficient, they can be linearized. Supposing that we have a generic nonlinear term as *max* $(x_1, x_2, x_3, \ldots, x_n)$, it can be converted to an equivalent linear structure by introducing a new positive variable y and a set of binary variables z_i, and by adding the following constraints:

$$max(x_1, x_2, x_3, \ldots, x_n) \rightarrow y \tag{24}$$

$$y \geq x_i \qquad \forall i = 1, \ldots, n \tag{25}$$

$$y \leq x_i + M \times z_i \qquad \forall i = 1, \ldots, n \tag{26}$$

$$\sum_{i=1}^{n} z_i \leq n - 1 \tag{27}$$

In the above, M is an arbitrarily large number. Constraints (25) state that y should be greater than all x_i, as y is the maximum of x_i. Constraints (26) and (27) ensure that at least for a single *i, y* must be lower than or equal to x_i, to prevent *y* from approaching infinity. It should be noted that when the objective function is "*minimization*", the constraints (26–27) are not necessary, but when the objective function is "*maximization*" and/or the model structure is "*multi-objective*", these constraints are compulsory.

Moreover, we have some bilinear terms as $Z_{jv}T_{jv}$ and $Y_{if}Y_{iffjrm}$ in the model, and these terms are explicitly nonlinear. Without loss of generality, suppose that we have a bilinear term as x.z, where x is a positive variable and z is a binary variable. Again, by introducing a new positive auxiliary variable y and adding the following constraints, it can be converted to an equivalent linear structure as follows:

$$x.z \rightarrow y \tag{28}$$

$$x - (1 - z) \times M \leq y \leq x \tag{29}$$

$$y \leq M \times z \tag{30}$$

Again, M is an arbitrary large number.

4. Solving procedures

As discussed in the prior section, the investigated IPDS problem is formulated as a nonlinear bi-objective model that is converted to a linear bi-objective one. Most practical scheduling models naturally involve the problem of simultaneous optimization of a number of objectives which might be in conflict with each other. Such problems, known as 'multi-objective optimization problems', aim to simultaneously optimize several conflicting criteria. In contrast to single-objective models that generate a single optimal solution, multi-objective models give a set of optimal solutions, named a Pareto optimal set, or non-dominated solutions that dominate other solutions. Indeed, Pareto solutions are solutions which cannot improve one of the objectives without degrading at least one other objective (Deb, 2001).

Owing to the non-deterministic polynomial-time hardness (NP-hard) nature of both FJS scheduling problems (Garey et al., 1976; Kacem et al., 2002) and VRP (Dantzing and Ramsar, 1959), our proposed model would consequently also be NP-hard. To solve the proposed problem in small-scaled instances, various exact techniques from multi-objective decision making (MODM) methods can be utilized. Nevertheless, these methods are not able to solve large-sized problems within a reasonable time, pushing us to develop a hybrid meta-heuristic algorithm.

4.1. ε-constraint method

The bi-objective nature of the proposed IPDS model enables us to apply some MODM techniques to solve it for small-scaled instances. MODM techniques are categorized as *a priori*, *a posteriori*, and *interactive* methods. Under an *a priori* method, a multi-objective optimization model is converted to a single objective model, and a decision maker (DM) puts his/her preferences before solution procedure. In an *a posteriori* method, the DM selects his/her most preferred solutions from among a set of generated optimal solutions (Pareto set), and in the *interactive* method, the DM is involved in the search procedure, and his/her preferences influence the direction in which the feasible space is being explored (Mirzapour Al-e-hashem and Rekik, 2014, Mirzapour Al-e-hashem et al., 2019).

The ε-constraint method is a well-recognized *a posteriori* technique. In this technique, in each step, one of the objective functions is optimized, while others are added as the constraints of the model with upper bound ε, as follows:

$$
\begin{aligned}
& min\ f_1 \\
& \text{s.t} \\
& f_2 \leq \varepsilon_2 \\
& \vdots \\
& f_n \leq \varepsilon_n
\end{aligned}
$$

To solve a multi-objective optimization problem with the ε-constraint method, the following steps are required:

1. One of the objective functions is selected as a primary objective to be optimized, and the other objectives are converted into constraints of the model by considering ε as the upper bound for each of them.

2. Each objective function is optimized individually, and then the interval $I_i = (f_i^*, f_i^-)$ between the optimum and the worst values of the objective function $f_i (i = 2, \dots, n)$ is divided into a pre-specified number (m), and the values of $\varepsilon_2, \dots, \varepsilon_n$, are then calculated, accordingly.

3. The problem created in step 1 is solved several times, with different values of ε_i varying in the interval I_i, to deduce a set of Pareto solutions.

4.2. Hybrid Particle Swarm Optimization (HPSO)

As the investigated IPDS problem is NP-hard, it is crucial to develop heuristic or meta-heuristic algorithms to handle the problem for large-sized instances. Moreover, owing to the multi-objective framework of the model, a modified version of these algorithms is required. Thus, a hybrid approach, combining the ε −constraint method and PSO algorithm and capable of producing Pareto solutions, is proposed to solve the model in large (or medium-sized) instances. (PSO is a social-based evolutionary technique introduced by Kennedy and Eberhart in 1995 for solving continuous optimization problems. The basic idea for developing the theory of the particle swarm is driven from the social behaviors of animals like birds and fishes. The PSO algorithm is initialized with a population (or swarm in PSO) of random solutions, named particles, flying through the solution space with a velocity, and updating themselves by following the previous optimum particles. The process of the developed hybrid PSO (HPSO) algorithm is structured as the following steps:

1. Set counter = 1 (counter =1, …, MaxIt), m = 1 (m = 1, …, M, M is the size of the Pareto set, e.g., M=10) and initialize a population of N particles/solutions.

2. Set $\varepsilon = f_2^{min} + m \times \frac{(f_2^{max} - f_2^{min})}{M}$

3. for each $i = 1, \dots, N$:

3.1. Initiate the p_i (position of each particle).

3.2. Initiate the vel_i (speed of each particle), and set it equal to 0.

4. Compute the cost function of particle *i* by applying f_1, plus the penalty of total violations for the model constraints (3–23) and that belonging to the second objective function (f_2), which is already converted to the constraints with an upper bound ε and a pre-specified number *(m).*

5. Set the exiting best position of particle *i* ($pbest_i$) equal to the initialized position of particle *i*.

6. Find the best position of all particles in the population ($gbest$).

7. Update the velocity of particle *i* using the following equation:

$vel_i = w * vel_i + C_1 * rand_1 * (pbest_i - p_i) + C_2 * rand_2 * (gbest - p_i)$, where C_1 and C_2 stand for the weights for moving a particle toward the best positions of individual particles and moving a particle toward the best position in the swarm, respectively. w is the inertia weight, and it represents the willingness of the particle to keep its velocity from the previous iteration. $rand_1$ and $rand_2$ are two random parameters in the range of [0,1].

8. Update the position of particle *i* by applying the following equation:

$p_i = p_i + vel_i$

9. Compute the cost function of particle i by applying f_1 plus the penalty of total violations for the model constraints (3–23), and that for the second objective function (f_2) converted to the constraints and restricted by a upper bound ε.

10. If the current value of objective function for each particle i is better than the $pbest_i$, then assign the current position of particle *i* to $pbest_i$.

11. If the current value of objective function for each particle *i* is better than *gbest*, then set the current position of particle *i* equal to *gbest*.

12. If counter= MaxIt, or any termination criteria are met, report the best solution and go to step 13, otherwise go to step 6.

13. If m=M stop, otherwise set m=m+1 and go to step 2.

5. Computational results

The aim of this section is fourfold: 1) to validate the applicability of the proposed model in small-sized problems, 2) to illustrate the implications of the integration contribution on both cost and customers, 3) to show Pareto optimal solutions and also that substantial conflict exists between the two given objectives, and 4) to examine performance of the proposed HPSO algorithm in medium-and large-sized problems.

5.1. Numerical illustration

Here, based on the data derived from one working day of the case study, our novel optimization model is implemented for a small-sized illustrative numerical example, to show the optimal configurations of production, vehicles, and routes under different preferences of the DM, and to depict a Pareto optimal set.

Assume the numbers of orders (n), operations for each job (R), processing machines (M), and available vehicles (V) as equal to 3, 3, 2, and 6, respectively. The variable transportation cost per minute (θ_v), the weights of earliness (μ) and tardiness (φ), the size of jobs (δ_j), and the processing cost on each machine (λ_m) are assumed to be equal to 1 (for all vehicles), 0.3, 0.7, [48, 36, 35], and [350,400], respectively. The time window for each order $[a_j, b_j]$ is set as $a_j = [70,100,190]$ and $b_j = [90,120,210]$. The data regarding capacity and the fixed cost of each vehicle are listed in Table 2. It is noteworthy that the capacity is given by an aggregate unit that is equivalent to a wooden part with approximate dimensions of 1.5×0.2×0.05 m^3, as the products are assembled in a customer zone, and the trucks often transport separated unassembled parts. In addition, information data regarding processing time and a process-machine matrix related to operation r of job (order) j on machine m are provided in Tables 3 and 4, respectively. The travel distances between customer i and j are provided in Table 5.

Table 2. *Vehicles Data*

Vehicle type v	Capacity	Fixed cost
1	90	180
2	80	200
3	100	100
4	70	165
5	105	120
6	85	150

Table 3. *Processing time of operation r of order j on machine m*

Machine	Job	1			2			3		
	Operation	1	2	3	1	2	3	1	2	3
1		10	8	6	12	6	11	12	7	8
2		7	9	11	7	10	8	6	11	10

Table 4. *Process-machine matrix*

Machine	Job	1			2			3		
	Operation	1	2	3	1	2	3	1	2	3
1		1	0	1	1	1	0	0	1	1
2		0	1	1	1	0	1	1	0	1

Table 5. *Transportation time between the customers*

Customer *j*	0	1	2	3
0	0	88	71	45
1	88	0	41	75
2	71	41	0	38
3	45	75	38	0

The proposed IPDS model and $\varepsilon-$constraint method are coded under the optimization programming language (OPL) and a CPLEX script, via the IBM ILOG CPLEX Optimization Studio 12.8.

We solved the proposed IPDS model for three different values of ε, and derived the optimal solutions for each one. The obtained values of two given objective functions, the total cost of production and distribution (f_1), and the weighted sum of delivery earliness and tardiness (f_2) are presented in Table 6. Details of the obtained results for the most important decision variables, i.e., production scheduling, distribution scheduling, and routing, are reported in Tables 7, 8, and 9.

Moreover, the assignment of the operations to each machine and the routes taken by the vehicles are schematically shown in Figures 2, 3, and 4, under the three different values of ε, respectively.

a) In a first situation, the DM aims to optimize the total scheduling cost without considering customer satisfaction (the weighted sum of delivery earliness and tardiness). The value of ε is equal to f_2^{max}, and the ε −constraint model used to generate the optimal solution is as follows:

Model P_1:

$$min f_1$$
s.t
Constraints (3–23)
$$f_2 \leq f_2^{max}$$

b) In the second solution of the Pareto set, the DM is concerned regarding the total scheduling cost three times more than regarding the customer concerns, and we used the following ε −constraint model, in which the value of ε is equal to $\frac{f_2^{max}-f_2^{min}}{4}+f_2^{min}$.

Model P_2:

$$min f_1$$
s.t
Constraints (3–23)
$$f_2 \leq \frac{f_2^{max}-f_2^{min}}{4}+f_2^{min}$$

c) In the last Pareto point, the DM aims to minimize the total scheduling cost, while the weighted sum of delivery earliness and tardiness is restricted to its best value (f_2^{min}). In this situation, ε is equal to f_2^{min}, and the applied ε −constraint model is as follows:

Model P_3:

$$min f_1$$
s.t
Constraints (3–23)
$$f_2 \leq f_2^{min}$$

Table 6*. Optimal solutions of two given objective functions for different values of ε*

	$\varepsilon = f_2^{max}$	$\varepsilon = \frac{f_2^{max}-f_2^{min}}{4}+f_2^{min}$	$\varepsilon = f_2^{min}$
f_1 (total scheduling cost)	25460	25500	26720
f_2 (tardiness/earliness)	103.7	51.3	21

Table 7. Optimal solutions of the ε-constraint method for $\varepsilon = f_2^{max}$

Production sub problem				Distribution sub problem					
$\pi_{j,r}$	Value	$\Upsilon_{j,r}$	Value	S_v	Value	E_v	Value	D_j	Value
π (3,1)	0	Υ(3,1)	6	S(3)	21	E(3)	111	D(1)	173
π (3,2)	6	Υ(3,2)	13	S(5)	61	E(5)	261	D(2)	132
π (3,3)	13	Υ(3,3)	21					D(3)	66
π(1,1)	21	Υ(1,1)	31						
π(1,2)	31	Υ(1,2)	40						
π(1,3)	40	Υ(1,3)	46						
π(2,1)	40	Υ(2,1)	47						
π(2,2)	47	Υ(2,2)	53						
π(2,3)	53	Υ(2,3)	61						
Production cost (f_{11})			24950$	Distribution cost (f_{12})				510$	

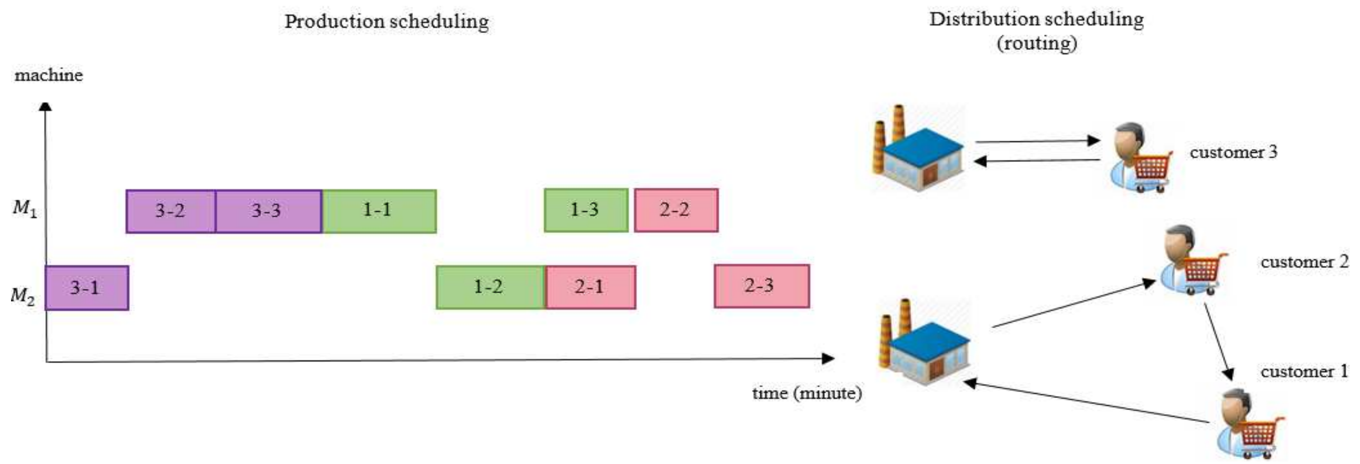

***Figure 2.** Schematic results of the ε −constraint method for $\varepsilon = f_2^{max}$*

In the first solution of the Pareto set (Table 7), in which the DM concentrates on cost and ignores customer satisfaction, none of the orders are delivered in their committed delivery time window (see data in section 6.1). Moreover, according to Figure 2, under this value of ε ($\varepsilon = f_2^{max}$), the majority of the orders are processed on machine 1, which has the lowest processing cost, and the total time of the routes taken by the vehicles ($\sum_{v=1}^{V}(E_v - S_v)$) equals 290 minutes, as the lowest possible time for distributing the orders in this example. Indeed, the DM is enabled to act according to his/her priority to merely optimize cost, without regarding customer concerns. More importantly, the pickup times and configurations of production, vehicles, and routes are not designed to deliver finished products by their given deadliness. For example, as the deadline of the order 1 has the earliest due date, it should be processed and the delivered first of all; alternatively, order 3 could be processed and delivered last to avoid early delivery.

Table 8. *Optimal solutions of the ε-constraint method for* $\varepsilon = \frac{f_2^{max} - f_2^{min}}{4} + f_2^{min}$

Production sub problem				Distribution sub problem					
$\pi_{(j,r)}$	Value	$\Upsilon_{(j,r)}$	Value	$S_{(v)}$	Value	$E_{(v)}$	Value	$D_{(j)}$	Value
$\pi(1,1)$	0	$\Upsilon(1,1)$	10	S(3)	29	E(3)	205	D(1)	117
$\pi(1,2)$	13	$\Upsilon(1,2)$	22	S(5)	37	E(5)	191	D(2)	120
$\pi(1,3)$	23	$\Upsilon(1,3)$	29					D(3)	82
$\pi(2,1)$	0	$\Upsilon(2,1)$	7						
$\pi(2,2)$	10	$\Upsilon(2,2)$	16						
$\pi(2,3)$	22	$\Upsilon(2,3)$	30						
$\pi(3,1)$	7	$\Upsilon(3,1)$	13						
$\pi(3,2)$	16	$\Upsilon(3,2)$	23						
$\pi(3,3)$	29	$\Upsilon(3,3)$	37						
Production cost (f_{11})			24950$	Distribution cost (f_{12})					550$

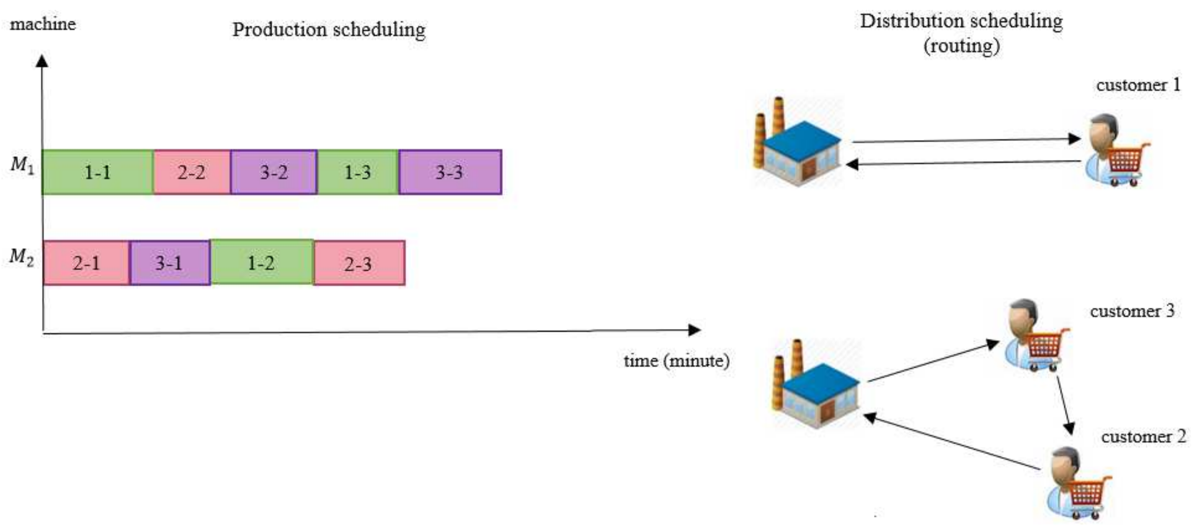

Figure 3. *Schematic results of the ε-constraint method for* $\varepsilon = \frac{f_2^{max} - f_2^{min}}{4} + f_2^{min}$

In this solution, the DM is concerned with customer satisfaction to some extent (its weight is one third of the cost). Indeed, here, the DM respects the customer due dates more than in the previous scenario. Following that and according to Figure 3, order 2 is delivered in its given delivery time-window whereas orders 1 and 3 are not delivered on time, and the total time of the routes taken by the vehicles increases from 290 to 330 minutes, meaning that the model is working accurately in accordance with the DM setting his/her preferences.

Table 9. *Optimal solutions of the ε-constraint method for $\varepsilon = f_2^{min}$*

Production sub problem				Distribution sub problem					
$S_{(j,r)}$	Value	$C_{(j,r)}$	Value	$St_{(v)}$	Value	$Et_{(v)}$	Value	$Dt_{(j)}$	Value
S(3,1)	0	C(3,1)	6	St(3)	32	Et(3)	240	Dt(1)	120
S(3,2)	10	C(3,2)	17	St(5)	40	Et(5)	182	Dt(2)	111
S(3,3)	22	C(3,3)	32					Dt(3)	195
S(1,1)	0	C(1,1)	10						
S(1,2)	13	C(1,2)	22						
S(1,3)	23	C(1,3)	29						
S(2,1)	6	C(2,1)	13						
S(2,2)	17	C(2,2)	23						
S(2,3)	32	C(2,3)	40						
Production cost (f_{11})		26150$		Distribution cost (f_{12})		570$			

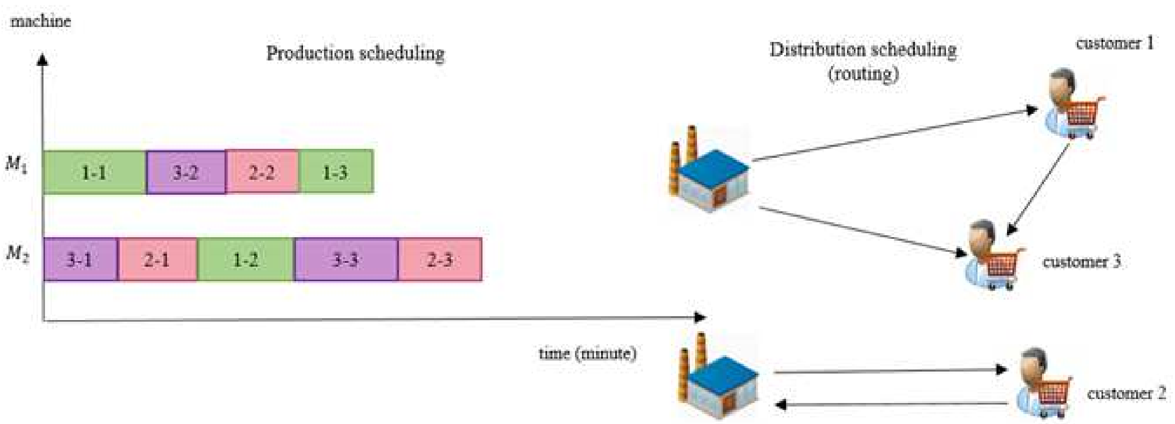

Figure 4. *Schematic results of the ε-constraint method for $\varepsilon = f_2^{min}$*

In the last solution of the Pareto set (Table 9), and conversely to the previous one, the DM aims to minimize the total operational cost, whereas the delivery earliness or tardiness is already optimized. Indeed, this situation is equivalent to one where the DM is only concerned with customer satisfaction. In more detail, Figure 4 depicts that in contrast to the first solution, orders 2 and 3 are delivered in their given delivery time-windows, and the majority of the operations are processed on machine 2, which has a higher processing cost but allows the system to deliver the products on time by avoiding bottlenecks. In addition, the total time of the routes taken by the vehicles increases from 330 to 350 minutes, which is the highest time for routes to be served.

We also note that according to Table 6, in the first solution of the Pareto set, the first objective function (minimizing the total production and distribution scheduling cost) equals 25460, and in the second and last solutions increases to 25500 and 26720, respectively. Additionally, in the first, second, and last solutions, the value of the weighted sum of delivery earliness and tardiness as the second objective function decreases as follows: 103.7 → 51.3 → 21. This simply confirms the existing conflict between the two objective functions.

Consequently, our framework allowed the DM to judgmentally choose the value of ε, and to integrate the customer concerns in his/her optimization of cost. Indeed, by enabling the integration option, our model permits the DM to judgmentally create a balance between the two conflicting objectives.

It is also worthwhile to show the Pareto set of optimal solutions, which allows the DM to select the most preferred solutions according to his/her preferences, and it shows the substantial conflict that exists between cost and customer concerns. Figure 5 depicts the Pareto optimal solutions, wherein by decreasing the weighted sum of delivery earliness and tardiness, there is an increase in the total cost of production and distribution.

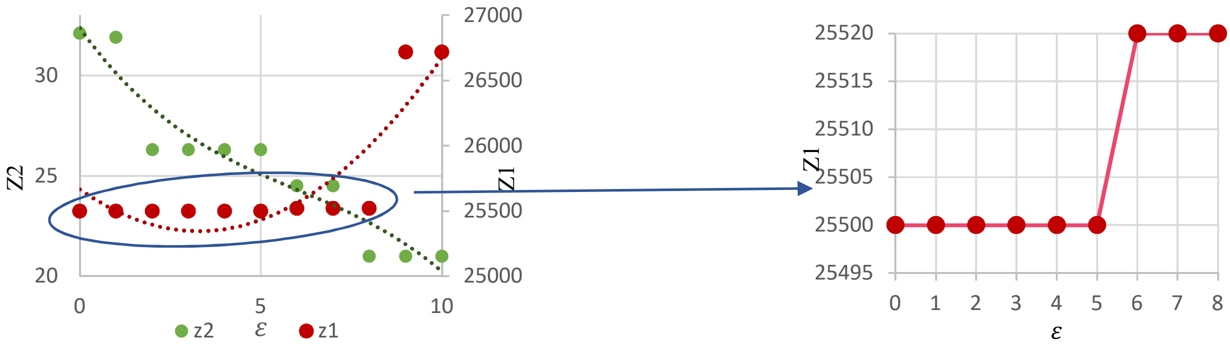

Figure 5. *Total cost of production and distribution (f_1) against the weighted sum of delivery earliness/tardiness (f_2)*

5.2. Performance of HPSO

In this section, we conduct a computational experiment to evaluate the performance of the proposed metaheuristic algorithm. In particular, 15 test problems taken from the applied case study in small, medium, and large sizes are solved by a proposed HPSO, and then the obtained results are compared with the optimal solution (or lower bound) of CPLEX. The proposed HPSO is coded in MATLAB R2015a, and all computations are run on a PC with 2.2 GHz and 7.89 GB RAM under Microsoft Windows 7.

In each case of small, medium, and large size instances, 5 test problems are generated, in which the variable delivery cost per minute is $\theta_j = 1$, and the weights of earliness and tardiness are $\mu = 0.3$ and $\varphi = 0.7$, respectively. The size of the orders, the delivery time window, the shipping time between customers, and the processing time of each operation of each job are drawn from $\delta_j \in [10,\ 60]$, $a_j = [70{,}300]$, $b_j = a_j + 20$, $t_{ij} \in [10\ \ 100]$, and $P_{jrm} \in [6\ \ 12]$, respectively. The capacity and the fixed cost of the vehicles are respectively drawn from $Q_v \in [50\ 200]$ and $F_v \in [100\ 200]$, and finally, the processing cost of each machine (λ_m) is randomly generated between [300 700].

Each test problem is solved for two points of the Pareto set under models P_1 and P_2, where the value of ε is equal to f_2^{max} and f_2^{min}, respectively. In this manner, all test problems are first solved by the ε −constraint method via CPLEX and then are solved by the developed HPSO algorithm, and the obtained results are reported in Table 10. In Table 10, the GAP_1 (in percentage) is used to calculate the percentage deviation of the average first objective function (total cost of production and distribution) obtained by HPSO (f_1^{HPSO}) from the one obtained by CPLEX (f_1^{LB}), and is calculated as follows:

$$GAP_1 = \frac{f_1^{HPSO} - f_1^{LB}}{f_1^{LB}} \times 100$$

In addition, GAP_2 (in percentage) is the deviation of the average of the second objective function (weighted sum of delivery earliness and tardiness) obtained by HPSO (f_2^{HPSO}) from the one obtained by CPLEX (f_2^{LB}), and its formulation is as follows:

$$GAP_2 = \frac{f_2^{HPSO} - f_2^{LB}}{f_2^{LB}} \times 100$$

It should be noted that CPLEX could not generate a model to solve some of the larger-sized problems. Moreover, for some test problems, CPLEX could not solve the proposed model in under two hours, and we used the lower bound of the problem as reported by the CPLEX to calculate the gaps.

As seen in Table 10, the HPSO algorithm converges to optimal solutions in less than eleven minutes for all medium-sized problems (P_1- P_6). In addition, for large-scale problems (P_7-P_{15}), at the best case, the proposed algorithm could generate near-optimum solutions in approximately two hours, with minimum, maximum, and average gaps of 11.18%, 14.93%, and 13.81%, respectively, from the lower bound reported by CPLEX. As CPLEX cannot solve these problems optimally in a reasonable time, we rely on the best bound obtained (which is not a feasible solution) after two hours. This means that the real optimal gap is definitely smaller that the reported values. Based on the CPU time, the HPSO algorithm, on average, has a better performance for larger-sized problems. The differences between the lower bound (non-optimal solution) of the CPLEX software and the best solutions of the proposed HPSO are sufficiently small. When the results are reviewed, a mere 5% difference from the global optimum in medium-sized problems and an 11% difference from the lower bound in large-size problems is sufficient persuasion of the competency of the proposed algorithm's performance. These results prove the high potential of the proposed HPSO in achieving better solutions in acceptable times. Furthermore, in all small-sized instances and without any exceptions, the proposed HPSO algorithm attains the global optimum in a few minutes.

Table 10. *Performance of the developed metaheuristic*

Problem	Orders	Operations	Machines	Vehicles	CPLEX					HPSO				
					# variables	$f_1^{LB}*$	CPU time**	$f_2^{LB}*$	CPU time**	$f_1^{HPSO}*$	$f_2^{HPSO}*$	CPU time	GAP_1	GAP_2
P_1	2	2	2	6	478	12300	00:00:00:26	11.9	00:00:00:36	12425	12.2	00:00:26:62	1.02	2.52
P_2	2	3	3	6	914	19450	00:00:00:44	14	00:00:00:47	20605	14.35	00:00:47:11	5.94	2.5
P_3	3	3	2	6	1126	25460	00:00:01:11	21	00:00:09:04	25913	21.29	00:00:52:72	1.78	1.38
P_4	3	3	3	10	1720	24545	00:00:01:65	16.7	00:01:21:76	24569	17.2	00:00:68:26	0.10	2.99
P_5	4	3	3	10	2506	37310	00:00:06:58	21	00:32:20:33	37535	22.2	00:02:41:06	0.60	5.71
P_6	5	3	3	10	3440	49483	00:04:04:09	61.3	02:00:00:00	50929	63.98	00:10:85:57	2.92	4.37
P_7	7	3	4	12	7380	104247	01:36:26:40	14.4	02:00:00:00	119000	16.45	02:00:00:00	14.15	14.23
P_8	8	3	4	12	9158	77684	02:00:00:00	108.7	02:00:00:00	89960	123.98	02:00:00:00	12.81	14.05
P_9	9	3	5	12	13054	102843	02:00:00:00	193.6	02:00:00:00	115199	215.26	02:00:00:00	12.01	11.18
P_{10}	10	3	3	15	11940	169270	02:00:00:00	131.54	02:00:00:00	191700	151.18	02:00:00:00	13.25	14.93
P_{11}	12	3	2	15	13075	-	00:53:15:39***	223	02:00:00:00	200240	255.37	02:00:00:00	-	14.51
P_{12}	12	3	3	15	19486	210200	02:00:00:00	360.79	02:00:00:00	234011	410.95	04:00:00:00	11.328	13.90
P_{13}	15	3	4	18	30742	N/A	03:00:00:00	N/A	03:00:00:00	1567100	2062900	04:00:00:00	-	-
P_{14}	20	3	5	22	63986	N/A	03:00:00:00	N/A	03:00:00:00	3375200	4288400	05:00:00:00	-	-
P_{15}	25	4	7	25	193999	N/A	03:00:00:00	N/A	03:00:00:00	9792400	9755100	05:00:00:00	-	-

* Solving the first objective function (f_1) is equal to solving model p_1 and also solving the second objective function (f_2) is equal to solving model p_2.

** For medium and large-scale problems, the lower bounds (LB) of CPLEX reported after two hours are used for comparison.

*** In this test problem, CPLEX is stopped as "out of memory", and until the reported time, could not generate the model to solve it.

N/A: CPLEX cannot reach a feasible solution after three hours.

6. Value of integration

In this section, we investigate the importance of the joint scheduling of production and distribution, through comparing the integrated model proposed in this study and the hierarchal schedules of these two operations. In this context, four test problems for different numbers of jobs (n= 3, 4, 5) are provided for solving with both integrated and separate approaches, and the results are reported in Table 11.

In the separate approach, the production model is solved first without considering distribution constraints, with the objective of minimizing the production cost, $f_{11} = \sum_{j,r,m} \lambda_m P_{jrm} X_{jrm}$, to obtain the production cost and completion time of orders. As the departure time of the vehicles is equal to the completion time of the last job in each shipment, the obtained completion times from the production model are set as the parameters in the distribution model. Then, the distribution model with the objective function of minimizing the weighted sum of delivery earliness and tardiness, $f_2 = \varphi \times \sum_{j=1}^{N} max\left(D_j - b_j, 0\right) + \mu \times \sum_{j=1}^{N} max\left(a_j - D_j, 0\right)$, is solved, to obtain the total time of earliness and tardiness to measure customer satisfaction. To derive the distribution cost in the separate approach, we solve the distribution model with the objective function of distribution cost, $f_{12} = \sum_{v=1}^{V} [F_v W_v + \theta_v (E_v - S_v)]$.

To compare the results of the separate approach with the integrated method, we solve the proposed IPDS model twice in a single objective framework. First, we minimize the total cost of production and distribution scheduling $f_1 = \sum_{j=1}^{N} \sum_{r=1}^{R_j} \sum_{m=1}^{M} \lambda_m P_{jrm} X_{jrm} + \sum_{v=1}^{V} [F_v W_v + \theta_v (E_v - S_v)]$ to determine the values of the production and distribution costs and completion time of orders. Second, we minimize the weighted sum of delivery earliness and tardiness to evaluate the improvement on customer satisfaction.

Table 11. *Value of the integration*

		Separate approach					Integrated approach					
Problem	N	Distribution cost	Production cost	f_2	Total cost	Completion time	Distribution cost	Production cost	f_2	Total cost	Completion time	f_2 improvement(%)
1	3	510	41000	73.5	41510	(0,45,106,84,0)	510	41000	44.1	41510	(0,100,61,39,0)	40
2	5	658	63800	60.7	64458	(0,45,67,100,133,154,0)	658	63800	35.8	64458	(0,148,22,70,103,37,0)	41
3	3	510	63500	72.1	64010	(0,51,103,148,0)	510	63500	57	64010	(0,148,97,45,0)	20
4	4	525	100250	174.9	100775	(0,51,237,115,173,0)	525	100250	48.3	100775	(0,51,173,221,109,0)	72

As illustrated in Table 11, for all test problems, when production and distribution schedules are integrated, the weighted sum of delivery earliness and tardiness decreases, whereas the production and distribution scheduling costs are fixed. Such an improvement in the customer satisfaction with a minimum total scheduling cost implies that the integration policy brings a significant added value to the consumers, while keeping the solution as economic as possible. The rationale behind this observation is that the integration option permits reduction of the completion time of orders as much as possible. Hence, the integration of the two decisions brings a higher customer satisfaction level for products with urgent fulfillment constraints, which constitutes an important decision-making exercise for MTO businesses, as compared with the separate production-distribution optimization (Table 11).

The company studied herein had already established a traditional production scheduling system to manage shop floor issues. Their system firstly generated a production schedule, and then the department of transportation hierarchically derived the transportation schedule based on the given production completion times. After applying the new method successfully based on the integrated optimization approach, and despite the natural primary resistance of the company to applying new procedures, it could demonstrate the usefulness of such integration methodology. In that regard, in the medium term, their margin was raised by approximately 15 percent on average, and they observed up to a 20 percent improvement in time window violations. That means that the integration policy acts as a key for producing received orders as soon as possible and delivering finished products in the batch immediately after production to avoid violations to the committed delivery times. This was impossible without permitting the production system to flexibly choose the different process routes, and to consequently reduce the completion times through omitting the bottlenecks. More importantly, the integrated model is capable of improving customer satisfaction by designing an efficient optimization scheme, where production scheduling is performed based on delivery scheduling and imposed delivery time windows, pickups are fixed according to completion times, and routes are arranged along with given deadlines, all under the minimal total scheduling cost. If the integration option is not enabled, i.e., according to the company's prior experience with the old method, the customized products are usually not delivered within their committed time windows, resulting in a meaningful shift in early or late delivery. Figure 6 depicts the comparison of the weighted sum of delivery earliness and tardiness obtained in the separate approach, versus that obtained in the proposed integrated model.

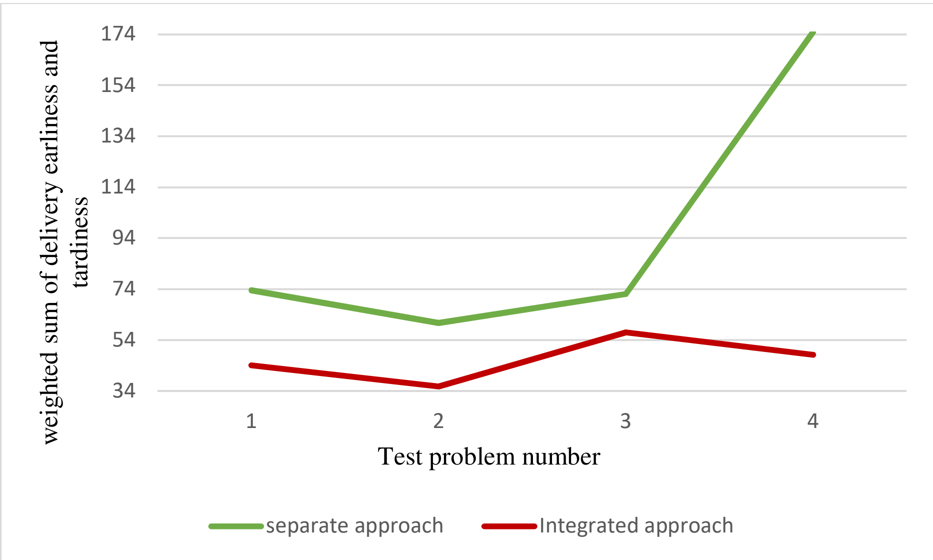

Figure 6. *Comparison of the weighted sum of delivery earliness and tardiness under the integrated and separate approaches*

As illustrated in Figure 6, as compared with the separate approach, the integrated production-distribution optimization leads to a significant decrease of the total time of earliness and tardiness. As a summary, and according to the Table 11 and Figure 6, joint scheduling is advantageous from both customer satisfaction and economic points of view. Practically, our framework provides a suitable tool for enabling a company to adopt an integration strategy as a lever not only to achieve the highest level of customer satisfaction, but also to operate as a coordinator between economic and customer criteria.

7. Conclusion and suggestions for future research

This study presents a bi-objective mixed integer model for managing an integrated FJS scheduling-vehicle routing problem with time window (VRPTW) constraints in an MTO business. The model finds a joint optimization scheme between production and distribution scheduling decisions, such that the trade-off between total operational cost and the weighted sum of delivery earliness and tardiness is optimized.

Inspired from real case study and based on data extracted from an FJS-based manufacturer, the proposed model is first optimally solved with the ε −constraint method. Then, to address medium- and large- sized problems in a reasonable time, an efficient HPSO algorithm is developed, and it is validated by solving an extensive set of test problems. The results showed that our framework can permit the company to create a balance between conflicting criteria: cost and customer time window satisfaction. We particularly discussed a situation where the integration policy can act as a lever, not only to improve customer satisfaction through reducing the total completion times by allowing flexible process routes (resulting in faster pickups), but also to help the company keep the total production and distribution scheduling costs at the minimum possible levels.

To the best of our knowledge, the current study is among the first investigations studying a flexible machine scheduling problem by considering transportation decisions. In addition to filling the research gaps, our study concerns a practical routing delivery problem with a *soft time-window*. The applicability of the developed framework is also enabled by allowing the use of different orders ‘sizes and a heterogeneous fleet composed of vehicles with different capacities, and fixed and variable costs.

Modeling the problem as a multi-objective optimization ensures an added value from theoretical and practical point of views. The developed model combines and trade-off two conflicting performance measures in a bi-objective modeling framework: cost minimization, and customer satisfaction maximization. Such a conflicting impact of decisions is naturally a challenging concern for decision makers in a highly competitive environment. From a theoretical point of view, we model our problem as a generic bi-objective model. Thus, our model could be adapted to solve other production and/or distribution systems with simpler machine (such as flow-shop) and/or delivery configurations (such as split delivery). From a managerial point of view, we demonstrated to practitioners the feasibility of the integration between production and distribution and more importantly we showed that it leads to positive impact on both the economic and customer service performance indicators.

For future research, we suggest studying the integration of production scheduling with an inventory routing problem (IRP) for vendor-managed inventory systems, where the customers are retailers rather than end users, and their inventory levels are managed by the supplier (manufacturer). Indeed, moving from a business-to-consumer (B2C) to a vendor-managed inventory (VMI)-based business-to-business (B2B) supply chain structure adds additional cost components (such as overstock and understock penalties), and stock control would be part of the trade-offs addressed in the present study. It would also be interesting to study the joint production-distribution scheduling problem under an uncertain environment caused, for instance, by random yields and random machine failures.

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