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A Complex networks based analysis of jump risk in equity returns: An evidence using intraday movements from Pakistan stock market

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Short running title Complex Networks and Jump Risk in Pakistan stock market

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A Complex networks based analysis of jump risk in equity returns: An evidence using intraday movements from Pakistan stock market

Abstract

We employ a multi-stage methodology combining complex network analytics and financial risk modelling to unveil the correlation structures amongst the price jump risks of companies forming the KSE-100 index in Pakistan. We identify the most influential companies in terms of jump risk, and identify communities- clusters of companies with similar price movement characteristics or with highly correlated price jumps. We find that equities in Pakistan stock market experience jumps in different time periods that are correlated to varying degrees within and across industries resulting in 19 different communities, four of which are strongly connected. While Oil & Gas, Cement and Banking sectors exhibit a significant representation of firms in communities, the automobile industry, however, seems to play an important role in risk propagation. These results provide an interesting insight to investors and other stakeholders from an emerging market viewpoint identifying the major sectors driving the volatility of KSE-100 index.

Keywords: Complex Network Analysis; Intraday Returns; Realised Jumps; Realised Volatility; Jump Risk

Jel codes : C10; C22; C58; G10

I. Introduction

Modelling stock market volatility and jump risk has gained significant attention in the fields of financial theory and practice. Recent developments in financial modelling and econometrics are focusing on techniques that deal with return and volatility simultaneously. For an investor, a correctly harnessed risk is vital to reduce risk factors and to generate solid returns. Stock price jump risk is one of the relevant phenomena that tend to enhance correlations amongst stocks. Volatility and jump risk are vital for asset allocation, derivative pricing, portfolio development and financial risk management. Since the publication of the seminal paper by Mantegna (1999) on the hierarchical structure of financial markets, several analytical and computational advancements in complex network analysis as well as financial modelling of price volatility and price jump risks have contributed to a better understanding of the stock markets' dynamics. So far, several papers have analysed realised volatility and/or realised jump and correlations among realised price volatility or realised price jumps of stocks listed on developed stock markets such as European and North American stock markets (Karanasos et al. 2020; Kremer, 2017; Hansen et al. 2012; Andersen et al., 2011; Eraker, 2004; Engle 2002), but the literature lacks a study that focuses on realised price jumps of stocks listed on emerging markets like Pakistan whose intraday data is not easily available. This study attempts to fill this void and adopts a nonparametric approach to extract realised jumps in stock prices, reveal correlation structures, and estimate the clustering effects among these realised jumps using a multi-stage methodology combining the fields of financial risk analytics and complex network analytics.

An asset price process experiences infrequent jumps in addition to the continuous Brownian motion and drift movements (Andersen et al., 2007). Jump risk is the sudden and frequent movement of large magnitude in the prices of securities. When there is a rapid substantial change in a stock's price, a jump is said to have occurred. The jump risk often occurs in the presence of an unusual event like the 2008 Financial Crisis; however, it can also be expected during normal periods of financial markets as well. The financial disruptions of US stock market crash (1987), Asian Currency Crisis (1997) and Great Financial Crisis (GFC-2008) had significant effects on security markets around the world and disrupted the functioning of other foreign stock markets. Besides foreign financial markets' linkages, stocks' prices are more prone to the news and signals propagating from the local micro and macro economies as well as the political environment. Many small-scale events of the local environment such as liquidity problems in the market, non-favorable implications of the policies' changes by State Banks and/or Governments, uncertain political environment, and unclear policies and regulations may render jump risk associated with stocks' prices higher in magnitude and frequency. The price jumps generate risks for investors regardless of whether they price risk positively (Driessen and Maenhout, 2013) or negatively (Cremers et al., 2015). The jumps behavior of volatility is quite different from normal and time varying volatility. The modeling frameworks of volatility in the stock market can be classified into two broad categories (Giovannini and Jorion, 1989, Bakshi et al., 1997); namely, time-varying volatility models - ARCH (Engle, 1982) and GARCH (Bollerslev, 1986) type of models, and Jump risk models in which quadratic variation (price process) is considered as a measurement tool for diffusion risk (Merton and Morton, 1980).

Historically, numerous research studies have used daily frequency of stocks' prices to study the jump risk factor despite the inherent limitation that the daily data do not capture the volatility fluctuations throughout a day. However, recent developments in the fields of Computing and Big Data have made it possible to record prices with higher frequencies. The data of intraday level and even on the frequency of a minute's level provide a whole new source of information and inference. By utilising fast computing techniques, recent studies have used these high frequency data to uncover new characteristics, behavior, and other aspects relating to the volatilities of securities. Of high significance are the contributions on volatility modelling by, for example, Andersen and Bollerslev (1998) who decomposed Deutsche Mark-Dollar Volatility into intraday activity patterns, macroeconomic announcements, and volatility persistence (ARCH) known from daily returns, Andersen (2001) who proposed one of the first measures of realised volatility that use intraday data, Barndorff-Nielsen and Shephard (2004) who proposed the bipower variation (BPV) measure and a multipower variation (MPV) measure to estimate integrated variance, and Andersen et al. (2012) who proposed a measure known as MinRV to estimate integrated variance. On the other hand, according to stock network theory, the stocks in the same market are closely linked; thus, they follow identical high or low patterns simultaneously. This underlying behavior in the stock market has been termed as 'stock networks'. With the advancements in computing, several algorithms have been developed to analyse stock networks. In this paper, we propose a multi-stage methodology for analysing price jump risk and its potential contagion effect in the Pakistani stock market, as proxied by the KSE-100 index, over the period ranging from September 12, 2018 to May 31, 2019.

This paper contribution to the literature is manifold. It is the first attempt to analyse the jump risk of stocks forming the Pakistani stock market index KSE-100 using high-frequency data on stock prices. Pakistan's stock market remained a red hot in recent years. In 2017, the KSE index witnessed an impressive surge to 500% since 2009 and left the neighbouring markets in China and India far behind. Since then, the market has been experiencing a turbulent phase. It touched a bottom around 27000 points (i.e., a decline of approximately -48%) in January 2020. In addition, using a multi-stage methodology combining financial risk modelling and complex network analytics, we unveil the correlation structures amongst the price jump risks of companies forming the KSE-100 index, identify the most influential companies jump risk-wise, and identify communities or clusters of companies with similar price movement characteristics or with highly correlated price jumps which are highly likely to experience a chain reaction after an influencer experiences a price jump.

The main findings of this study could be summarised as follows. Stock prices exhibit high volatility in different time periods and realised jump extraction results confirm the presence of jump risk in companies listed under KSE-100 Index. The jump risk is an exigent portion of stock price realised volatility (Chao et al., 2017; Guo et al. 2019). In addition, there are important correlations amongst price jumps of certain companies' stocks, as revealed by the minimum spanning tree (MST) structure of the complex network of stocks forming the KSE-100 Index; thus, stock price jumps in one stock will cause a price jump in neighboring stocks in the MST. Furthermore, nineteen communities of stocks were identified and reveal that, in the Pakistani stock market, two car manufacturers; namely, Pak Suzuki Motors and Indus Motors Company Limited (Toyota), lead the list followed by MCB Bank as the most influential in the MST, whereas companies like K-Electric and Nishat Mills Limited lying at the edge of the complex network have smaller influence; i.e., their price jumps are not correlated with those of other companies. The largest community consists of nine companies viz. UBL, HBL, Meezan Bank, PSO, Hascol, OGDCL, Sui Northern, Indus Motors and Kohat Cement Company Limited; whereas, the smallest community consists of two companies; namely, Engro Fertilizer and Pioneer Cement. Finally, some companies like MCB, Honda Atlas and Fauji Cement are not part of any community. Overall, sectors such as Oil & Gas, Cement, Banking, Engineering, Fertilizer and Automobile have a clearly dominant representation across communities such that Oil & Gas and Cement sectors which are represented in 7 communities, while Banks, Engineering, Fertilisers, and Automobile sector firms are observed in 4 communities each. The firms in the largest community are clustered around four major sectors, including Oil & Gas, Cement, Banking and Automobile sector. The significance of these sectors is apparent from the fact that the combined market capitalization of three sectors only, including Oil & Gas, Cement and Banking sector, is approximated around 15.80 Billion USD which is almost 34% of the total market capitalization of KSE-100 index. These trends may potentially provide a useful insight to investors and other stakeholders of the market, in particular, the sectors driving the market volatility.

The remainder of the paper is organised as follows. Section 2 describes the data and methodology used, section 3 presents the empirical findings while section 4 concludes the paper.

II. Data and Methodology

In this section, we shall provide details on our dataset (section 2.1) and describe our methodology (section 2.2) – see Figure 1 for a snapshot of the design of the methodological framework.

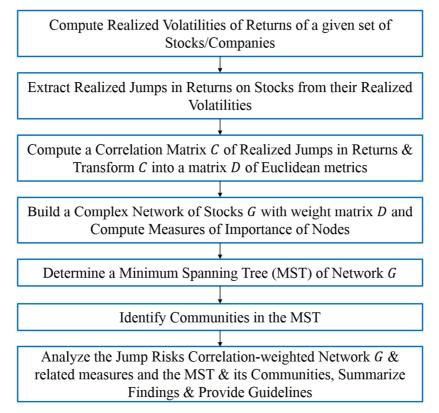


Figure 1: Methodology for analysing jump risk in a stock market

II.1. Data

For analysis purposes, this study uses intraday stock prices of 100 firms listed on Pakistan stock exchange and belong to the KSE-100 Index representing 34 sectors. The data consist of

925,641 transactions recorded from September 12, 2018 to May 31, 2019. Using the high frequency package of R, this transaction data has been categorised into 5-minute frequency and data on stocks with incomplete trading days was discarded. As a result of the cleaning and reorganisation of the initial data, we are left with a dataset of 168 days for 48 companies having 12,415 transactions.

II.2. Methodology

Hereafter, we shall describe in detail the steps of the proposed methodology for analysing jump risk in a stock market.

Step 1: Compute realised volatilities

For each stock i (i = 1, ..., n) and trading day t (t = 1, ..., T), first compute its *intraday returns* $r_{i,k}^t$ as follows:

$$r_{i,k}^{t} = \ln\left(\frac{p_{i,k}^{t}}{p_{i,k-1}^{t}}\right), k = 1, \dots, \tau$$

where $p_{i,k}^t$ denote the *k*th price of stock *i* on day *t*, $p_{i,k-1}^t$ denote the five min lag price on day *t*, τ denote the number of five-minute intraday return observations in a day *t*, and *T* denote the number of trading days in a year. Then, compute the *realised variance* on any day *t*, say RD_i^t , as follows:

$$RD_i^t = \sum_{k=1}^{\tau} r_{i,k}^{t\,2}, t = 1, \dots, T.$$

Finally, compute the *realised volatility* on any day t, say RV_i^t , as follows:

$$RV_i^t = \sqrt{T \cdot RD_i^t}, t = 1, \dots, T.$$

Step 2: Extract realised jumps from realised volatilities

A realised volatility (RV) estimator measures the total quadratic variation of the observed returns process, including the contribution from the cumulative squared jumps. On the other hand, an integrated variance (IV) estimator measures the continuous part of the quadratic variation. Thus, a RV estimator minus an IV estimator provides an estimate of realised jumps. Typical estimators of IV in the presence of jumps include the bipower variation (BPV) measure and multipower variation (MPV) measures (e.g., tripower variation measure, quadpower variation measure, fourth order power variation measure). In this paper, we use instead an estimator proposed by Andersen et al. (2012), known as MinRV, which provides additional robustness to jumps and/or market microstructure noise by using nearest neighbor truncation; that is, the absolute returns are truncated at a level controlled by the neighboring returns. To be more specific, MinRV uses one-sided truncation as each intraday return is compared only to the subsequent absolute return as reflected in its mathematical formulation:

$$MinRV_{i}^{t} = \frac{\pi}{\pi - 2} \left(\frac{\tau}{\tau - 1} \right) \sum_{k=1}^{\tau - 1} min\left(\left| r_{i,k}^{t} \right|, \left| r_{i,k+1}^{t} \right| \right)^{2}.$$

In sum, for each stock i (i = 1, ..., n) and trading day t (t = 1, ..., T), the *realised jump* of stock i on any day t, say RJ_i^t , is computed as follows:

$$RJ_i^t = RV_i^t - MinRV_i^t.$$

Step 3: Compute realised jumps' correlation matrix *C* and transform it into an Euclidean distance matrix *D*

The (i, j)-entry of the realised jumps' correlation matrix, say $C_{n \times n}$, is computed as follows:

$$corr(RJ_i, RJ_j) = \frac{\sum_{t=1}^T (RJ_i^t - \mu(RJ_i)) (RJ_j^t - \mu(RJ_j))}{\sqrt{\sum_{t=1}^T (RJ_i^t - \mu(RJ_i))^2} \sqrt{\sum_{t=1}^T (RJ_j^t - \mu(RJ_j))^2}},$$

where $\mu(RJ_i) = \sum_{t=1}^{T} RJ_i^t / T$. Since this realised jumps' correlation matrix is intended to be used for building a weighted network of *n* stocks, say *G*, in the next step where the weight matrix should satisfy the properties of an Euclidean distance matrix, on one hand, and correlation coefficients could be negative and therefore cannot be used as a distance between stocks, on the other hand, hereafter we shall use a generalised metric defined using as distance an appropriate function of the correlation coefficient. To be more specific, we transform the correlation coefficients $corr(RJ_i, RJ_j)$ into positive real number weights as follows:

$$d(i,j):corr(RJ_i,RJ_j) \in [-1,1] \rightarrow \sqrt{2\left(1-corr(RJ_i,RJ_j)\right)} \in [0,2].$$

We shall denote the distance matrix thus obtained by D = (d(i, j); i, j = 1, ..., n).

Step 4: Build a network of stocks G with weight matrix D and compute relevant measures to get topological insights

In order to analyse the relationships between the price jumps of n stocks, as measured by the realised jumps computed in step 2, a weighted complete graph G = (V, E, D) is constructed where V denote the set of vertices representing stocks or companies, E denote the set of edges connecting pairs of vertices or stocks, and D is the matrix of weights of edges representing the distances between vertices or equivalently the strengths of the relationships between jump risks of stocks, which are a function of the correlation coefficients. We shall refer to graph G as a realised jumps correlations-weighted network of stocks or simply a complex network.

Then, to get topological insights from complex network G, several measures are available in the social network analysis and complex network analysis literatures. However, for our application, the most relevant measures are vertex-related and belong to the centrality measures category, as the centrality of a vertex measures its potential importance, influence, or prominence in a network based on its relative position compared to other vertices in a network. In our case, influential stocks may influence investors' decisions during extreme events when some stocks or categories of stocks often tend to move together. Amongst centrality measures, betweenness and closeness centralities are especially enlightening the important roles some vertices could play in the network. *Closeness Centrality* of a given vertex i to all other vertices in the graph or undirected network, where the geodesic distance between two vertices is the number of edges on the shortest path linking them (Bavelas, 1950) and thus reflects how close vertex i is to all other vertices in the network:

$$C_C(i) = 1/\sum_{j \neq i} \#e(\pi_{ij});$$

where $#e(\pi_{ij})$ denote the number of edges on the shortest path π_{ij} between vertices *i* and *j*. Note that $C_C(i) \in [0,1]$. Note also that a vertex or stock *i* with higher $C_C(i)$ is closer to all other vertices in the network and $C_C(i)$ could be viewed as a measure of the 'speed' with which stock *i* price jump effect flows or propagates through the network (contagion speed). On the other hand, Betweenness Centrality of a given vertex *i*, say $C_B(i)$, measures how often a vertex appears on the shortest paths between two other vertices in the network (Freeman, 1977) and thus reflects the relative importance of a vertex in linking two other vertices through a shortest path in the network:

$$C_B(i) = \sum_{j \neq i} \sum_{k \neq i} \frac{\# \pi_{jk}(i)}{\# \pi_{jk}},$$

where $\#\pi_{jk}$ denote the number of shortest paths between vertices *j* and *k*, and $\#\pi_{jk}(i)$ denote the number of shortest paths between vertices *j* and *k* that pass through vertex *i*. Note that $C_B(i)$ is often normalised as follows to obtain a measure that lies within the interval [0,1]:

$$\frac{C_B(i) - \min_{\substack{j \in V}} C_B(j)}{\max_{\substack{j \in V \\ j \in V}} C_B(j) - \min_{\substack{j \in V \\ j \in V}} C_B(j)}$$

Note also that a vertex or stock i with higher $C_B(i)$ would have wider price jump effect over the network of stocks (contagion spread), as it is directly or indirectly connected to other stocks by lying on a higher proportion of the shortest paths between them.

Step 5: Determine a Minimum Spanning Tree (MST) of complex network G

Trees, in general, and spanning trees, in particular, play an important role in the investigation of the dynamical and topological properties of complex networks. In edge-weighted networks such as complex network G = (V, E, D), a spanning tree of G is a subgraph of G which covers all the vertices of G with the minimum possible number of edges. A minimum spanning tree (MST) of G is a spanning tree of G which has the minimum total weight out of all possible trees that span the entire network G. Note that a spanning tree for a complete graph with n vertices has n-1edges; thus, any edge belonging to the MST indicates that the jump risks of the corresponding stock prices are connected or relatively highly correlated. As compared to the correlation network, or equivalently the correlation matrix C, the MST provides a skeletal structure with only n-1 edges, and thus attempts to strip the system's complexity down to its essentials (Johnson et al, 2005). As shown by Mantegna (1999), the practical justification for using the MST lies in its ability to provide economically meaningful information. Since the MST contains only a subset of the information from the correlation matrix, in principle it cannot tell us anything which we could not obtain by analysing the matrix C itself. However, it turns out that often it can provide an insight into the system's overall behavior which would not be so readily obtained from the (large) correlation matrix itself. Note that the number of spanning trees for a complete graph with n vertices is n^{n-2} ; thus, it would be computationally inefficient to consider all the possible spanning trees to derive the MST. Instead, there are three widely used algorithms for deriving a MST, i.e., Kruskal's algorithm (Kruskal, 1956), Prim's algorithm (Prim, 1957) and Sollin's algorithm (Sollin, 1965). In this paper, we use Prim's algorithm, because our complex network is dense (Jarvis & Whited, 1983). Prim's algorithm builds a MST one vertex at a time, starting from an arbitrary vertex in G, at each step adding the cheapest possible connection from the tree to another vertex not yet in the tree until all vertices are in the tree.

Step 6: Identify communities in the MST

In many applications, complex networks are characterised by a heterogeneous structure (e.g., a heterogeneous distribution of the weights of the edges connecting vertices) which often results in the presence of a so-called community structure, where a network, say G = (V, E, D), exhibits a

community structure if and only if its set of vertices V can be divided into two or more (potentially overlapping) subsets – also referred to as clusters – so that the vertices belonging to the same subset or cluster are similar (e.g., densely connected; have strong links; have common features) and those belonging to different subsets or clusters are not (e.g., not connected or sparsely connected; have no links or weak links; do not have much features in common). In practice, the existence of communities in a complex network generally affects various processes such as the spreading of the effect of a specific event over the network; therefore, it is important to find out whether communities exist within correlation network G. In addition, communities often have very different properties compared to the average properties of the network, which might be misleading.

In order to identify communities or clusters in a network; such as our complex network G, a variety of clustering algorithms are available. These algorithms could be classified into one of several categories based on the type of approach they adopt for determining communities: (1) modularity-based approach; e.g., FastGreedy (Clauset et al., 2004), Louvain (Blondel et al., 2008), Leading eigenvector (Newman, 2006) and RenEEL (Guo et al., 2019); (2) node similarity-based approach; e.g., WalkTrap (Pons & Latapy, 2005); (3) compression-based approach; e.g., InfoMap (Rosvall et al., 2007, 2010); (4) significance-based approach; e.g., Order Statistics Local Optimization Method (Lancichinetti et al., 2011); (5) diffusion-based approach; e.g., Girvan & Newman algorithm (Girvan & Newman, 2002) and Visualization Of Similarities or VOS for clustering (Waltman et al., 2010). In this paper, we choose to use the VOS for clustering algorithm to uncover communities, because of its attractive design features as outlined below.

The aim of VOS for clustering is to find for each vertex i a positive integer x_i that indicates the cluster or community to which vertex i belongs so that stocks or companies that have a high similarity are located close to each other, whereas companies that have a low similarity are located far from each other. To be more specific, the objective to optimise/minimise is:

$$V(x_1, \dots, x_n) = \sum_{\substack{i < j \\ i, j \in E(MST)}} s_{ij} \delta_{ij}^2 - \sum_{\substack{i < j \\ i, j \in E(MST)}} \delta_{ij}$$

where E(MST) denotes the set of edges in the MST, δ_{ij} is given by

$$\delta_{ij} = \begin{cases} 0 \ if \ x_i = x_j \\ \frac{1}{\gamma} \ if \ x_i \neq x_j \end{cases}$$

the association strength s_{ii} of vertices *i* and *j* is given in our case by

$$s_{ij} = d(i,j) = \sqrt{2\left(1 - corr(RJ_i, RJ_j)\right)},$$

and γ ($\gamma > 0$) is a parameter referred to as the resolution parameter and represents the degree of 'resolution' of analysis. Note that $V(x_1, ..., x_n)$ can be interpreted in terms of attractive and repulsive forces between vertices, where the first term represents an attractive force and the second term represents a repulsive force. Thus, the overall effect of the two forces is that vertices with a high association strength are pulled towards each other while vertices with a low association strength are pushed away from each other. Note also that, by means of the resolution parameter γ , VOS for clustering can deal with the resolution limit problem (Fortunato & Barthélemy, 2007) experienced by, for example, modularity-based clustering methods which may fail to identify small clusters; in sum, VOS for clustering can always identify small clusters by choosing a sufficiently large value for γ .

Step 7: Analyse the jump risks correlation network G and related measures, its MST and related communities, and summarise findings and provide guidelines. The outcome of this step, including plots of the MST and its communities, is detailed in the next section. Hereafter, we shall describe the VOS method used for mapping entities (e.g., stocks or companies) in a highdimensional space into a lower-dimensional space for visualisation purposes (van Eck & Waltman, 2007). The aim is to find for each vertex i (i.e., object or entity such as a stock or company) a vector of coordinates $x_i \in \mathbb{R}^2$ that indicates the location of vertex *i* in a 2dimensional map so that stocks or companies that have a high similarity (i.e., their realised price jumps are highly correlated) are located close to each other, whereas companies that have a low similarity are located far from each other. This is achieved by minimising the following objective function:

$$V(x_1, \dots, x_n) = \sum_{\substack{i < j \\ i, j \in E}} s_{ij} d_{ij}^2 - \sum_{\substack{i < j \\ i, j \in E}} d_{ij}$$

where d_{ij} denotes the distance between vertices i and j and is given by

$$||x_i - x_j|| = \sqrt{\sum_{k=1}^2 (x_{ik} - x_{jk})}$$

and the association strength s_{ij} of vertices *i* and *j* is given by $s_{ij} = \sqrt{2(1 - corr(RJ_i, RJ_j))}$ in

our case.

III. **Empirical Findings**

III.1. Intraday Return, Realised Volatility and Realised Jump

In this section, the statistical properties of intraday returns, realised volatility and realised jumps are reported – see Tables 1-3.

As an example, the time series of five-minute intraday returns of OGDCL is plotted in Figure 2 thereby confirming the fluctuations of five-minute returns of the company. The average returns of Pak Suzuki Motors are the highest (0.0050) and those of Millat Tracttors Limited are the lowest (-0.0290). The Indus Motor Company Limited has the highest maximum return value of 113.79 and K-Electric has the lowest maximum value of 0.42 while Mari Petroleum Company Limited has the lowest minimum value of -142.71 and K-Electric has the highest minimum value of -0.39. Furthermore, Mari Petroleum Company Limited has the highest standard deviation value of 4.8263 and K-Electric has a lowest value of 0.0224.

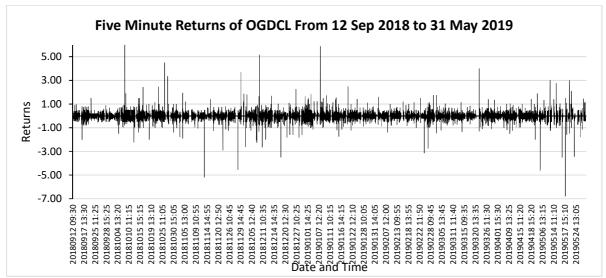


Figure 2: 5-Minute High-Frequency Return of OGDCL

As reported in Table 2, Mari Petroleum Company Limited has the highest average volatility of 296.1223 and K-Electric has the lowest average volatility of 1.4546. Figure 3 shows the intraday volatility of OGDCL and it is clear that there are low and high volatility patterns in five-minute prices.

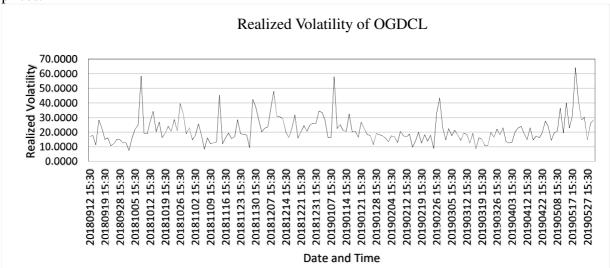


Figure 3: 5-Minute High-Frequency Realised Volatility of OGDCL

The results of realised jumps are different than those of realised volatility. K-Electric has highest average value of 6.2088 and Lucky Cement has lowest average value of 0.1373. Figure 4 shows the movements of 5-minute realised jumps of OGDCL for September 2018 to May 2019. We can observe relatively large price jumps in September 2018, November 2018 and March 2019. Also, there have been relatively smaller jumps during the specified period indicating that the market is not stable. However, despite some companies showing little jumps, overall the KSE-100 Index exhibits realised jumps.

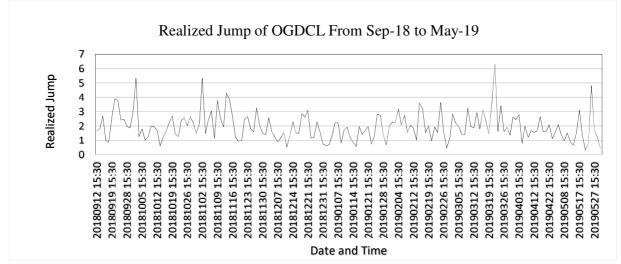


Figure 4: 5-Minute High-Frequency Realised Jumps of OGDCL

| Name | Minimum | Maximum | Mean | Standard Deviation | Skewness | Kurtosis |
|---------------------------|----------|---------|---------|-----------------------|----------|-----------|
| OGDCL | -6.8000 | 6.0000 | -0.0012 | 0.3328 | 0.1703 | 63.2265 |
| Pak Electron | -1.4200 | 2.4600 | -0.0009 | 0.1173 | 1.3469 | 42.3962 |
| Pak Intl Bulk | -0.9500 | 0.7700 | -0.0003 | 0.0407 | -0.1052 | 84.1025 |
| Pak Petroleum | -31.0000 | 8.5000 | -0.0033 | 0.5690 | -13.8479 | 762.6454 |
| PSO | -57.5500 | 16.9100 | -0.0113 | 0.8968 | -20.8632 | 1423.1459 |
| Pak Suzuki Motors | -28.4700 | 69.1000 | 0.0050 | 1.2567 | 14.4892 | 847.7936 |
| Pioneer Cement | -4.3700 | 2.7500 | -0.0012 | 0.1891 | -0.3138 | 57.9612 |
| Searle Co. | -40.3000 | 24.8900 | -0.0129 | 1.0392 | -3.5934 | 243.9521 |
| Sui Northern | -7.0100 | 4.9000 | -0.0017 | 0.2954 | 0.3055 | 64.5452 |
| Sui Southern | -1.2600 | 1.1500 | -0.0007 | 0.0883 | 0.4903 | 26.2244 |
| TRG | -1.8900 | 1.4400 | -0.0011 | 0.1076 | -0.1680 | 38.3230 |
| UBL | -8.5000 | 8.0000 | 0.0000 | 0.4617 | 0.6488 | 59.4464 |
| Attock Refinery | -41.7800 | 8.1800 | -0.0094 | 0.7069 | -16.5335 | 1014.2831 |
| BOP | -0.7400 | 1.0600 | 0.0000 | 0.0425 | 2.0707 | 73.9961 |
| D.G Khan Cement | -6.3900 | 5.2400 | -0.0034 | 0.3520 | 0.1763 | 53.6942 |
| Engro Corporation | -42.3900 | 15.0400 | -0.0024 | 0.8645 | -9.4626 | 517.1596 |
| Engro Fertiilizer | -4.5000 | 3.2400 | -0.0006 | 0.1761 | -1.2737 | 88.4767 |
| Fauji Cement | -1.3000 | 1.6400 | -0.0006 | 0.0810 | 1.0516 | 54.0435 |
| Hascol | -57.5100 | 16.1000 | -0.0163 | 0.8188 | -27.2766 | 2017.0507 |
| HBL | -8.5000 | 6.9000 | -0.0019 | 0.4293 | 0.1035 | 44.7779 |
| Honda Atlas | -17.2800 | 24.7800 | -0.0086 | 1.0835 | 1.3825 | 65.2114 |
| Hub | -4.9600 | 4.4200 | -0.0008 | 0.2485 | -0.8085 | 53.8627 |
| Int Steel | -4.2500 | 4.5700 | -0.0046 | 0.2938 | 0.2278 | 43.4500 |
| K-Electric | -0.3900 | 0.4200 | -0.0001 | 0.0224 | 0.7039 | 38.5579 |
| Lucky Cement | -26.7500 | 25.2100 | -0.0064 | 1.4583 | 0.7683 | 61.9339 |
| Maple Leaf | -2.6000 | 3.0000 | -0.0022 | 0.1842 | 0.5481 | 40.9384 |
| MCB | -5.3900 | 9.4500 | -0.0017 | 0.4955 | 1.8567 | 42.8404 |
| Nishat Chunian Ltd. | -3.0500 | 2.8300 | -0.0010 | 0.1791 | 0.0707 | 38.9482 |
| Fauji Fertilizer Co. Ltd. | -5.3600 | 8.8800 | -0.0002 | 0.2377 | 4.3528 | 238.4424 |
| Cherat Cement Co. Ltd. | -5.0000 | 3.4600 | -0.0036 | 0.2926 | -0.3325 | 40.2005 |
| Amreli Steels Ltd. | -5.0000 | 3.4600 | -0.0036 | 0.2926 | -0.3325 | 40.2005 |
| Kot Addu Power Co. | -6.9000 | 2.9500 | -0.0015 | 0.1422 | -8.8494 | 504.2026 |

| Ltd. | | | | | | |
|-------------------------------------|-----------|----------|---------|--------|----------|-----------|
| National Refinery Ltd. | -14.6400 | 15.4800 | -0.0164 | 1.0146 | -0.1710 | 50.5923 |
| Nishat Mills Ltd. | -7.5700 | 7.4400 | -0.0024 | 0.4480 | 0.1146 | 39.0190 |
| Engro Foods Ltd. | -4.3200 | 4.8200 | -0.0025 | 0.2849 | 1.0846 | 42.4441 |
| International Industries Ltd. | -12.5000 | 10.6400 | -0.0092 | 0.6775 | -0.1339 | 55.1710 |
| Bank Alfalah Ltd. | -6.0000 | 3.9500 | -0.0007 | 0.1791 | -2.5997 | 170.1191 |
| Millat Tractors Ltd. | -54.0000 | 49.0000 | -0.0289 | 3.0571 | 0.4841 | 62.0841 |
| PakistanTelecommunicat ion Co. Ltd. | -0.4400 | 0.5700 | -0.0002 | 0.0338 | 1.1324 | 40.7750 |
| Kohat Cement Co. Ltd. | -29.3500 | 5.1100 | -0.0051 | 0.4767 | -18.7483 | 1187.1048 |
| Mari Petroleum Co. Ltd. | -142.7100 | 85.0000 | -0.0286 | 4.8263 | -1.8641 | 126.2316 |
| Indus Motor Co. Ltd. | -79.9000 | 113.7900 | -0.0274 | 4.2531 | 1.2027 | 96.5239 |
| Attock Cement Pakistan Ltd. | -27.3800 | 5.8200 | -0.0049 | 0.4978 | -13.6641 | 772.7361 |
| Meezan Bank Ltd | -10.2500 | 5.0000 | -0.0002 | 0.3614 | -2.7277 | 126.3173 |
| GlaxoSmithKline | -7.0000 | 8.0000 | -0.0039 | 0.5275 | 0.8737 | 48.3580 |
| Pak Oil Field | -137.7300 | 15.9800 | -0.0163 | 1.7205 | -42.7235 | 3370.6095 |
| NBP | -2.1200 | 2.6500 | -0.0008 | 0.1381 | 1.4138 | 45.5724 |
| Fauji Fertilizer Bin Qasim Ltd. | -2.2500 | 1.8100 | -0.0017 | 0.1328 | 0.3523 | 41.6291 |

Table 2: Summary statistics of realised volatility

| Name | Minimum | Maximum | Mean | Standard Deviation | Skewness | Kurtosis |
|---------------------|---------|----------|---------|-----------------------|----------|----------|
| OGDCL | 7.5189 | 64.0898 | 21.5646 | 9.3152 | 1.7895 | 4.5059 |
| Pak Electron | 2.5569 | 20.9853 | 7.6954 | 3.4008 | 1.2042 | 1.9015 |
| Pak Intl Bulk | 0.0000 | 8.2661 | 2.1586 | 1.9702 | 0.4694 | -0.4173 |
| Pak petroleum | 6.3645 | 267.2986 | 33.6752 | 22.8536 | 6.8809 | 66.1361 |
| PSO | 0.0000 | 490.1062 | 48.6475 | 40.6914 | 7.9795 | 83.5674 |
| Pak Suzuki Motors | 0.0000 | 186.5191 | 39.0333 | 49.2723 | 0.8732 | -0.4701 |
| Pioneer Cement | 2.5863 | 38.5880 | 12.5166 | 5.1973 | 1.7024 | 5.0610 |
| Searle Co. | 0.0000 | 345.1306 | 63.4883 | 37.5463 | 3.3214 | 19.8240 |
| Sui Northern | 6.0887 | 62.8047 | 19.5993 | 7.9769 | 1.6629 | 4.9158 |
| Sui Southern | 2.1281 | 12.0246 | 5.9630 | 1.9331 | 0.8426 | 0.7013 |
| TRG | 2.1483 | 17.0189 | 7.0812 | 3.0037 | 1.0884 | 1.0997 |
| UBL | 4.3283 | 83.3479 | 29.7971 | 13.8445 | 1.2978 | 1.7486 |
| Attock Refinery | 0.0000 | 357.6248 | 39.7951 | 31.5557 | 6.3640 | 61.3961 |
| BOP | 0.6350 | 9.3972 | 2.6728 | 1.3917 | 1.8741 | 4.7431 |
| D.G Khan Cement | 7.0228 | 64.3482 | 22.8257 | 10.7227 | 1.1438 | 1.2445 |
| Engro Corporation | 8.9623 | 363.2156 | 51.2348 | 33.7684 | 5.2336 | 43.4853 |
| Engro Fertiilizer | 3.3973 | 38.8728 | 11.3941 | 5.4109 | 1.9866 | 5.5986 |
| Fauji Cement | 0.4948 | 14.3770 | 5.3071 | 2.3403 | 1.3060 | 2.0055 |
| Hascol | 6.6131 | 491.5666 | 41.8148 | 41.2238 | 8.0994 | 85.4619 |
| HBL | 5.2479 | 73.5384 | 28.3411 | 11.4688 | 1.4863 | 3.4538 |
| Honda Atlas | 0.0000 | 210.2653 | 69.9284 | 32.5587 | 1.3582 | 3.8008 |
| Hub | 0.0000 | 44.7155 | 16.0604 | 7.2794 | 1.3746 | 3.0703 |
| Int Steel | 0.0000 | 50.3045 | 19.2743 | 8.4772 | 1.0495 | 1.2659 |
| K-Electric | 0.0000 | 3.5425 | 1.4546 | 0.5532 | 1.3234 | 2.9260 |
| Lucky Cement | 11.4915 | 253.6916 | 94.7199 | 44.3599 | 1.1292 | 1.1588 |
| Maple Leaf | 3.9363 | 30.7444 | 12.0727 | 5.1797 | 1.1180 | 1.1956 |
| MCB | 6.0000 | 75.0772 | 31.5525 | 14.0840 | 1.0815 | 0.7672 |
| Nishat Chunian Ltd. | 0.0000 | 38.7002 | 11.6586 | 5.0931 | 1.4153 | 4.7344 |

| Fauji Fertilizer Co. Ltd. | 4.1500 | 76.6961 | 14.3618 | 9.2660 | 2.7893 | 12.6964 |
|---|---------|-----------|----------|----------|---------|----------|
| Cherat Cement Co. Ltd. | 4.5513 | 48.8614 | 19.2535 | 8.2016 | 1.0318 | 1.4604 |
| Amreli Steels Ltd. | 4.5513 | 48.8614 | 19.2535 | 8.2016 | 1.0318 | 1.4604 |
| Kot Addu Power Co. Ltd. | 0.0000 | 59.1225 | 8.2474 | 5.7212 | 4.7558 | 37.4103 |
| National Refinery Ltd. | 0.0000 | 218.0695 | 64.4356 | 32.4171 | 1.1147 | 2.5849 |
| Nishat Mills Ltd. | 0.0000 | 99.5442 | 29.1603 | 13.1250 | 1.6407 | 5.2101 |
| Engro Foods Ltd. | 6.4844 | 44.7914 | 19.0026 | 7.2341 | 0.9898 | 0.9838 |
| International Industries Ltd. | 0.0000 | 138.6933 | 42.9355 | 22.5155 | 1.4595 | 3.3168 |
| Bank Alfalah Ltd. | 0.0849 | 52.5482 | 11.1473 | 6.3377 | 2.5566 | 12.2749 |
| Millat Tractors Ltd. | 0.0000 | 612.5535 | 185.6056 | 112.1323 | 1.1459 | 1.1214 |
| Pakistan Telecommunication Co. Ltd. | 0.4243 | 5.2725 | 2.1550 | 1.0093 | 0.9775 | 0.6924 |
| Kohat Cement Co. Ltd. | 0.0000 | 250.7116 | 26.8902 | 20.8481 | 7.5878 | 79.7757 |
| Mari Petroleum Co. Ltd. | 0.0000 | 1414.5538 | 296.1223 | 179.7830 | 2.2030 | 9.3791 |
| Indus Motor Co. Ltd. | 0.0000 | 965.5402 | 254.4105 | 164.6525 | 1.1319 | 1.9624 |
| Attock Cement Pakistan Ltd. | 3.7470 | 237.0212 | 28.9123 | 21.1460 | 6.0042 | 55.9476 |
| Meezan Bank Ltd | 0.0000 | 86.9950 | 21.0920 | 13.7257 | 1.6343 | 4.8023 |
| GlaxoSmithKline | 0.0000 | 84.2243 | 32.9246 | 17.1639 | 0.6958 | 0.3739 |
| Pak oil Field | 21.9012 | 1174.7470 | 81.7760 | 92.1508 | 10.2004 | 119.9952 |
| NBP | 3.5527 | 33.6817 | 8.7983 | 4.2075 | 1.9921 | 7.4736 |
| Fauji Fertilizer Bin Qasim Ltd. | 1.2728 | 20.8526 | 8.6670 | 3.7644 | 0.8491 | 0.7461 |

Table 3: Summary statistics of realised jumps

| Name | Minimum | Maximum | Mean | Standard Deviation | Skewness | Kurtosis |
|-------------------|---------|---------|--------|-----------------------|----------|----------|
| OGDCL | 0.3359 | 6.2912 | 1.9585 | 0.9788 | 1.3131 | 2.8710 |
| Pak Electron | 1.4684 | 6.2963 | 5.0627 | 1.2838 | -1.2763 | 0.5580 |
| Pak intl bulk | 3.1324 | 6.2972 | 6.0242 | 0.4372 | -4.8808 | 26.0110 |
| Pak petroleum | -0.1045 | 3.0198 | 1.0512 | 0.5249 | 0.5975 | 0.4746 |
| PSO | 0.0099 | 3.4733 | 0.6969 | 0.4728 | 1.9517 | 7.4265 |
| Pak Suzuki Motors | -0.2816 | 1.4951 | 0.2921 | 0.4138 | 1.0820 | 0.8564 |
| Pioneer Cement | 0.6229 | 6.2334 | 3.1088 | 1.3330 | 0.5976 | -0.1487 |
| Searle Co. | -0.2547 | 6.2539 | 0.4973 | 0.6033 | 6.0732 | 53.5837 |
| Sui Northern | 0.4013 | 6.3064 | 2.1620 | 1.0977 | 1.2206 | 2.2121 |
| SUI Southern | 2.0133 | 6.2968 | 5.3097 | 1.1241 | -1.2506 | 0.4235 |
| TRG | 1.7978 | 6.2940 | 5.0571 | 1.2821 | -1.0236 | -0.3986 |
| UBL | 0.0056 | 6.3067 | 1.2097 | 0.7437 | 2.4595 | 12.9211 |
| Attock Refinery | -0.0586 | 4.1163 | 0.9943 | 0.6819 | 1.2439 | 3.0289 |
| BOP | 3.7313 | 6.2997 | 6.0856 | 0.3460 | -4.9946 | 26.9990 |
| D.G Khan Cement | 0.2759 | 6.3068 | 1.8440 | 1.0655 | 1.1255 | 1.7436 |
| Engro Corporation | -0.0803 | 1.7535 | 0.5432 | 0.3653 | 0.8325 | 0.8567 |
| Engro Fertiilizer | 0.6665 | 6.3064 | 3.3527 | 1.4217 | 0.5171 | -0.4172 |
| Fauji Cement | 1.6974 | 6.3036 | 5.7082 | 0.7983 | -2.5843 | 6.7317 |
| Hascol | 0.0281 | 6.3069 | 1.1674 | 1.1809 | 2.9959 | 9.9778 |
| HBL | 0.0273 | 3.2180 | 1.1884 | 0.5882 | 0.6603 | 0.7349 |
| Honda Atlas | -0.1410 | 3.7034 | 0.4606 | 0.4833 | 2.6227 | 13.0837 |
| Hub | 0.4942 | 6.3057 | 2.2438 | 1.3148 | 1.4699 | 2.1543 |
| Int Steel | 0.3562 | 6.3066 | 2.0353 | 1.1068 | 1.4562 | 3.0534 |
| K-Electric | 4.8897 | 6.3026 | 6.2088 | 0.1090 | -10.8025 | 131.0926 |

| Lucky Cement | -0.3673 | 0.8566 | 0.1373 | 0.2388 | 0.6181 | 0.3414 |
|---|---------|--------|--------|--------|---------|---------|
| Maple Leaf | 0.4511 | 6.2910 | 3.5982 | 1.4384 | 0.0612 | -0.9125 |
| MCB | -0.0584 | 4.6112 | 1.0086 | 0.6811 | 1.9775 | 7.0393 |
| Nishat Chunian Ltd. | 0.5401 | 6.3068 | 3.3158 | 1.4225 | 0.4498 | -0.3993 |
| Fauji Fertilizer Co. Ltd. | 0.4338 | 6.3065 | 2.9184 | 1.6243 | 0.8346 | -0.1984 |
| Cherat Cement Co. Ltd. | 0.2852 | 6.3063 | 2.0125 | 1.2707 | 1.9490 | 4.0001 |
| Amreli Steels Ltd. | 0.2852 | 6.3063 | 2.0125 | 1.2707 | 1.9490 | 4.0001 |
| Kot Addu Power Co. Ltd. | 0.4816 | 6.3056 | 4.2115 | 1.6465 | -0.3107 | -1.0460 |
| National Refinery Ltd. | -0.1177 | 6.3070 | 0.6391 | 1.0914 | 4.2239 | 18.9127 |
| Nishat Mills Ltd. | 0.0758 | 6.3069 | 1.4529 | 1.1907 | 2.3546 | 6.5708 |
| Engro Foods Ltd. | 0.2870 | 6.3069 | 2.1837 | 1.4721 | 1.5174 | 1.7518 |
| International Industries Ltd. | -0.0479 | 6.3070 | 1.0391 | 1.2549 | 3.2367 | 10.8714 |
| Bank Alfalah Ltd. | 0.6495 | 6.3070 | 3.8039 | 1.7311 | 0.1651 | -1.2174 |
| Millat Tractors Ltd. | -0.3448 | 6.3070 | 0.2669 | 0.8795 | 6.2715 | 41.1527 |
| Pakistan Telecommunication Co. Ltd. | 2.4688 | 6.3063 | 6.1559 | 0.4678 | -5.8714 | 36.8124 |
| Kohat Cement Co. Ltd. | 0.1610 | 6.3069 | 1.7861 | 1.6895 | 1.7774 | 2.2337 |
| Mari Petroleum Co. Ltd. | -0.2324 | 6.3071 | 0.4171 | 1.4454 | 3.7087 | 12.5007 |
| Indus Motor Co. Ltd. | -0.3077 | 6.3071 | 0.2784 | 1.1494 | 4.7682 | 22.3442 |
| AttockCementPakistan Ltd. | -0.0173 | 6.3070 | 1.6464 | 1.6225 | 1.9175 | 2.9327 |
| Meezan Bank Ltd | 0.0752 | 6.3070 | 2.2926 | 1.9362 | 1.1887 | 0.0495 |
| GlaxoSmithKline | -0.0280 | 6.3067 | 1.5357 | 1.6529 | 1.8675 | 2.5780 |
| Pak oil field | -0.3019 | 1.6704 | 0.3232 | 0.3039 | 1.3038 | 3.7328 |
| NBP | 0.3157 | 6.3058 | 4.1779 | 1.7443 | -0.3034 | -1.1782 |
| Fauji Fertilizer Bin Qasim Ltd. | 0.4432 | 6.3068 | 4.1974 | 1.6667 | -0.2377 | -0.9462 |

III.2. Complex Network

Recall that, by design, a shorter (respectively, larger) distance between two vertices of our complex network G = (V, E, D) represents a stronger (respectively, weaker) correlation among their prices' jumps. Different centrality metrics have been computed to identify 'influential' nodes in network G. The complete list of stocks along with their industries, communities, betweenness centrality and closeness centrality is provided in Table 4.

With regards to the clustering of industries into communities, we observe a mixed pattern but with a clearly dominant representation of sectors including Oil & Gas, Cement, Banking, Engineering, Fertilizer and Automobile manufacturers among the communities formed. Out of the nineteen communities formed, four major communities consist of 4 or higher number of firms, while community 2 is the largest one with 9 firms from four major sectors including Oil & Gas, Cement, Banking and Automobile. It is also apparent from these statistics that Oil & Gas has the highest number of total firms (10 firms) present across all the communities, respectively, followed by Cement sector (8 firms) and banking sector (7 firms), while the Oil & Gas sector and Cement sector with their presence in 7 communities each leads in terms of representation in the number of communities, followed by Banks, Engineering, Fertilisers, and Automobile manufacturers present in 4 communities each. It is equally worth noting that these sectors are major market movers. For instance, the combined market capitalisation of three sectors only, including Oil & Gas, Cement and Banking sector, is approximated around 15.80 Billion USD, which is almost 34% of the total market capitalisation of KSE-100 index.

With respect to betweenness centrality, the two leading car manufacturers of Pakistan, Pak Suzuki Motors and Indus Motors Company Limited (Toyota), lead the list followed by MCB Bank. On the other hand, with respect to closeness centrality, Pak Suzuki Motors, Indus Motors Company Limited (Toyota) and MCB Bank lead the list.

| Name | Industry | Community | Betweenness | Closeness |
|-----------------------------|---------------------------------|-----------|-------------|-----------|
| BOP | Commercial Banks | 1 | 0.0000 | 0.1703 |
| Cherat Cement Co. Ltd. | Cement | 1 | 0.0000 | 0.1703 |
| Amreli Steels Ltd. | Engineering | 1 | 0.0000 | 0.1703 |
| Bank Alfalah Ltd. | Commercial Banks | 1 | 0.0000 | 0.1703 |
| Mari Petroleum Co. Ltd. | Oil & Gas | 1 | 0.2414 | 0.2043 |
| Attock Cement Pakistan Ltd. | Cement | 1 | 0.0000 | 0.1703 |
| Fauji Fertilizer Bin Qasim | Fertilizer | 1 | 0.0000 | 0.1703 |
| OGDCL | Oil & Gas | 2 | 0.0000 | 0.2554 |
| PSO | Oil & Gas | 2 | 0.0000 | 0.2554 |
| Sui Northern | Oil & Gas | 2 | 0.0000 | 0.2554 |
| UBL | Commercial Banks | 2 | 0.0000 | 0.2554 |
| Hascol | Oil & Gas | 2 | 0.0000 | 0.2554 |
| HBL | Commercial Banks | 2 | 0.0000 | 0.2554 |
| Kohat Cement Co. Ltd. | Cement | 2 | 0.0000 | 0.2554 |
| Indus Motor Co. Ltd. | Automobile Manufacturing | 2 | 0.5828 | 0.3406 |
| Meezan Bank Ltd | Commercial Banks | 2 | 0.0000 | 0.2554 |
| Fauji Fertilizer Co. Ltd. | Fertilizer | 3 | 0.0000 | 0.2260 |
| Kot Addu Power Co. Ltd. | Power Generation & Distribution | 3 | 0.0000 | 0.2260 |
| National Refinery Ltd. | Oil & Gas | 3 | 0.0842 | 0.2901 |
| Hub | Power Generation & Distribution | 4 | 0.0842 | 0.2327 |
| K-Electric | Power Generation & Distribution | 4 | 0.0000 | 0.1895 |
| Engro Foods Ltd. | Dairy | 4 | 0.0000 | 0.1895 |
| GlaxoSmithKline | Pharmaceuticals | 4 | 0.1221 | 0.2938 |
| Searle Co. | Pharmaceuticals | 5 | 0.0426 | 0.2866 |
| SUI Southern | Oil & Gas | 5 | 0.0000 | 0.2238 |
| Engro Corporation | Fertilizer | 6 | 0.0000 | 0.2238 |
| Pakistan Telecom Co. Ltd. | Technology & Communication | 6 | 0.0426 | 0.2866 |
| | Engineering | 7 | 0.0833 | 0.2901 |

Table 4: Complex Network Betweenness and Closeness results

| Nishat Mills Ltd. | Textile | 7 | 0.0000 | 0.1865 |
|-------------------------------|-------------------------------|----|--------|--------|
| Millat Tractors Ltd. | Automobile Manufacturing | 7 | 0.0426 | 0.2282 |
| Pak Suzuki Motors | Automobile Manufacturing | 8 | 0.8104 | 0.3917 |
| Nishat Chunian Ltd. | Textile | 8 | 0.0000 | 0.2831 |
| Pak oil field | Oil & Gas | 8 | 0.0000 | 0.2831 |
| NBP | Commercial Banks | 8 | 0.0000 | 0.2831 |
| Pak intl bulk | Transport | 9 | 0.0000 | 0.1942 |
| TRG | Technology & Communication | 9 | 0.3164 | 0.2831 |
| International Industries Ltd. | Engineering | 9 | 0.2951 | 0.2398 |
| Pioneer Cement | Cement | 10 | 0.0426 | 0.2866 |
| Engro Fertiilizer | Fertilizer | 10 | 0.0000 | 0.2238 |
| Pak petroleum | Oil & Gas | 11 | 0.0000 | 0.2831 |
| Attock Refinery | Oil & Gas | 12 | 0.0000 | 0.2831 |
| D.G Khan Cement | Cement | 13 | 0.0000 | 0.2831 |
| Fauji Cement | Cement | 14 | 0.0000 | 0.2831 |
| Honda Atlas | Automobile Manufacturing | 15 | 0.0000 | 0.2831 |
| Int Steel | Engineering | 16 | 0.0000 | 0.2831 |
| Lucky Cement | Cement | 17 | 0.0000 | 0.2831 |
| Maple Leaf | Cement | 18 | 0.0000 | 0.2831 |
| MCB | Commercial Banks | 19 | 0.4921 | 0.3672 |

A minimum spanning tree (MST) of complex network G is depicted in Figure 5. The vertices in the center have a stronger influence while the vertices lying at the edge of the tree have lesser influence. When the price of stronger influence vertices jumps, there are lots of jumps in the stock price associated with them, whereas when the price of weaker influence vertices jumps, the stock's price jumps are not substantial.

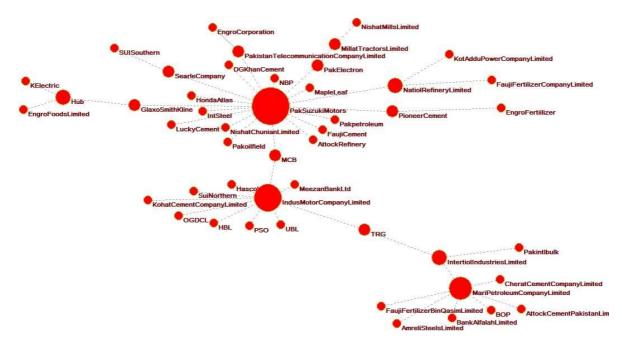


Figure 5: Realized Jumps Correlation Minimum Spanning Tree. Size represent the importance of the node in the MST

Finally, another important topological feature of complex network *G* being studied is the presence of communities. Vertices that are densely connected tend to be in the same community. The outcome is provided in Figure 6, which shows nineteen different clusters in the sample network with four major communities having four or more companies. The largest community is made up of nine companies including United Bank Ltd., Habib Bank Ltd., Meezan Bank, Pakistan State Oil, Hascol Oil, Oil and Gas Development Co. Ltd., Sui Northern Gas Ltd., Toyota Indus Motors, and Kohat Cement. The results reveal that jump correlation between these companies is high; i.e., a positive jump brings about a positive change in these connected companies. This potentially helps investors precisely forecast the firm-specific jumps to factor this in their investment and risk management strategies. More precisely, volatility jumps are indicative of strong correlations among stocks in the same community. Therefore, fluctuations in the prices of such stocks will cause the price of other stocks to fluctuate. The investor can manage the financial risk in light of the chain reaction within the same community.

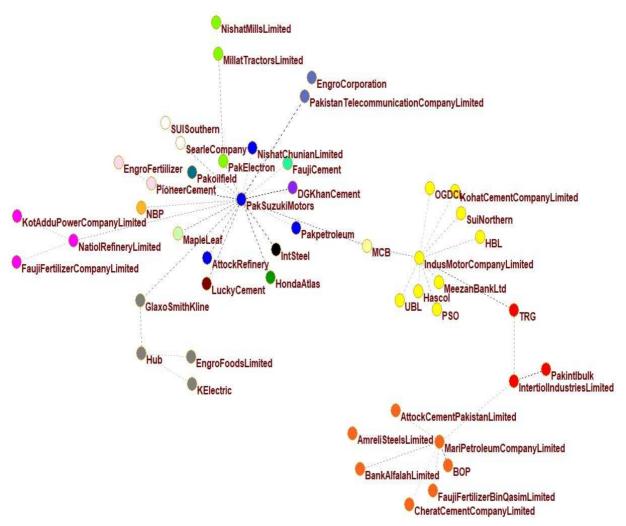


Figure 6: VOS Community Division Diagram

IV. Conclusions and Recommendations

In this study, a complex network of intraday KSE-100 index stocks, weighted by jump risk correlations, was built using intraday five-minute high-frequency stock price data from September 12, 2018 to May 31, 2019, and its vertices or stocks analysed by means of centrality measures. Analysis of vertices with the highest betweenness centrality as well as the highest closeness centrality suggest that the two leading car manufacturers of Pakistan (i.e. Pak Suzuki and Indus Motors-Toyota) along with MCB Bank lead the list. Then, a minimum spanning tree (MST) was determined using Prim's algorithm. The connected vertices (companies or their stocks) in the MST shows that a stock price jump in one stock will cause price jumps in other stocks, which are either directly or indirectly connected to it. Finally, the communities within the MST were identified using the VOS algorithm. The results suggest that the largest community is made up of nine companies including United Bank Ltd., Habib Bank Ltd., Meezan Bank, Pakistan State Oil, Hascol Petroleum Limited, Oil and Gas Development Co. Ltd., Sui Northern Gas Ltd., Toyota Indus Motors, and Kohat Cement. These results confirm that the car manufacturing industry is at the core of the whole industry. We suggest that the development of

the manufacturing industry would be vital to reduce the jump risk of the stock market of Pakistan. This study provides insight to investors about idiosyncratic jumps in the stocks and the correlations among jumps in a bid to better formulate their portfolio investment and risk management strategies.

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