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On the interplay between local lead times, overall lead time, prices, and profits in decentralized supply chains

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Abstract. A two-stage decentralized supply chain operates in make-to-order under a stochastic environment. Each stage represents an independent firm that quotes a price and a delivery time to its downstream while satisfying a minimum service level. The mean demand depends on the final price and the overall delivery time quoted to the customers by the whole supply chain. We study three settings. First, the downstream, as a Stackelberg leader, decides its price and controls both delivery times, and the upstream, as a follower, reacts by deciding its own price. Second, the downstream decides its delivery time and controls prices, and the upstream reacts by quoting its own delivery time. Third, the upstream, as a leader, decides its price and controls both delivery times, and the downstream, as a follower, decides its own price. This is the first study to investigate the delivery time quotation and pricing in decentralized supply chains where each firm performs operations and has a delivery time, and the demand is function of both upstream and downstream delivery times in addition to final price. We characterize analytically the optimal strategy under each setting and derive insights into the interplay between local delivery times, overall delivery time, prices, demand, and profits. We investigate how delivery times can be used to coordinate the supply chain and the impact of firms' capacities.

Keywords: Lead time; Pricing; Decentralized supply chains; Stackelberg game.

1 Introduction

Different industries like aerospace, automotive, construction, in vitro diagnostics, and computers production operate in make-to-order (MTO) in some stages of the supply chain (SC). MTO SCs can jointly use the DT and price as a strategic weapon to manage demand and increase profitability (Pekgün et al. 2017, Hafizoğlu et al. 2016, Huang et al. 2013). As demand and processing times are generally random in practice (Fang et al. 2013), the DT decision is

complicated since quoting a shorter DT to customers can yield a higher demand but increases the risk of late delivery. The decentralization of production activities, which characterizes most 21st-century SCs (Zhu 2015), requires to rethink the DT quotation and pricing decisions as it has led to SC configurations where operation stages are controlled and managed by independent firms, each one of them aims to maximizing its own profit (Warsing et al. 2019). Decentralization raises the question on how to coordinate the local DTs associated with firms' operations. As customers' demand depends on the overall DT guaranteed by the system, it does not suffice that each firm guarantees its own DT locally to its downstream, but the whole system must be able to guarantee the overall DT quoted to the customers (with a minimum service level). The relationship between local DTs and overall DT is also complicated by the stochastic nature of the problem. This study investigates the interplay between DT quotation, pricing, demand and profitability in decentralized MTO SCs when each firm undertakes operations (typically, manufacturing) and has a processing lead time.

We consider two independent profit-maximizing firms in an MTO SC where the mean demand depends on the final price and the overall DT quoted to end customers. Each firm has a finite capacity and a random processing time, and customers' demand is also random. We define this decentralized problem as a sequential game and study three decision settings. First, the downstream firm, acting as a leader, controls DT decisions and each firm decides its own price. Second, the downstream firm, acting as a leader, controls pricing decisions and each firm decides its own DT. Third, the upstream firm, acting as a leader, controls DT decisions and each firm decides its own price. These decision scenarios are motivated by practical cases as we explain in Sections 4, 5 and 6, respectively. Our contribution in this paper is twofold.

- This is the first study to investigate the DT quotation and pricing in decentralized MTO SCs where each firm performs operations and has a DT, and the demand is function of both upstream and downstream DTs in addition to final price. While most related papers studied the manufacturer-retailer SC where the retailer's DT is ignored or a supplier-manufacturer SC where the supplier's DT equals zero (as will be shown in Table 1), our model generalizes this framework by considering the DT of each firm and how it impacts the demand. Our model particularly fits with the case of SCs with different manufacturing firms where each firm undertakes operations. For instance, a manufacturer of semi-finished product who supplies a manufacturer of final product, or a machining firm who supplies an assembly firm in the manufacturing of industrial machines. Other relevant examples

include a design-engineering firm supplying a building firm in the construction industry. The existing models, either those considering a single firm under a centralized setting (e.g., Albana et al. 2018, Li et al. 2014, Zhao et al. 2012) or two independent firms but where the overall DT is equal to the DT of only one firm (e.g., Modak & Kelle 2019, Xiao & Qi 2016, Zhu 2015, Liu et al. 2007), cannot be used to study the interactions between each firm's DT and price decisions in the above practical cases. While the joint DT quotation and pricing in a stochastic environment is known to be a challenging problem even with a single firm, we obtain analytically the optimal solution for each firm and derive some interesting analytical insights, which we complement with numerical results.

- Our study reveals interesting trade-offs that govern the interactions between DTs, prices, demand, and profits, and leads to new managerial insights, some of them differ (or cannot be obtained) from existing models. For instance, we find that the local DTs are positively correlated whatever the decision scenario, which cannot be revealed by models where there is only one DT. Therefore, a firm will shorten its DT if and only if the other firm shortens its own DT. While related works found that the upstream and downstream prices are positively correlated (e.g., Liu et al. 2007, Xiao & Qi 2016), we show that this result does not always hold if both firms have operations and DTs, which means that a firm may have interest in decreasing its price if the other firm increases its own price. Another interesting result is that the follower's expected profit always increases in quoted DT and in mean demand, whereas the SC leader's expected profit is a concave function in quoted DT and in mean demand. This means that the follower has always interest in increasing demand while the SC leader will lose if demand exceeds a threshold limit. This result differs from the findings of Liu et al. (2007) and Xiao & Qi (2016) where the profits of both firms increase in demand. While most related studies consider only one SC leader (generally, the upstream), we compare different decision sequences (with either the upstream or the downstream as a leader) and show in case of balanced capacity (i.e., same capacity for both firms) that the SC will make the same overall profit and will offer the same product to the customers (in terms of price and DT) regardless of the firm who decides the DTs and acts as a leader. Our models consider that both firms have a finite capacity. We show that the amount of capacity allocated by each firm impacts the quality of SC coordination. For instance, when the SC leader controls the DTs and each firm decides its own price, selecting the firm with the largest capacity to act as a leader can generally improve the

SC coordination. Finally, we show under all settings that the local DTs can be effectively used to coordinate the SC. A judicious DT quotation can significantly increase the overall SC profit and even make it very close to the centralized profit. However, the coordination is less efficient in case of unbalanced production capacities.

A literature review is presented in Section 2. Section 3 describes the general modeling framework. In Sections 4, 5 and 6, we respectively motivate each setting and develop and solve the corresponding model. Analytical analysis and managerial insights will be discussed in Section 7. Section 8 is dedicated to the numerical experiments. Finally, we discuss the limitations and provide future research directions in Section 9.

2 Literature review

This study relates to the literature on joint DT quotation and pricing with DT- and price-sensitive demand. The relevant studies can be classified according to whether they consider a single firm, with typically a centralized decision process, or multiple firms that collaborate or compete under a decentralized decision process. We review the extant literature in both areas while focusing on papers that consider an MTO environment.

2.1 Single firm and centralized decision process

Most of the relevant studies are based on the framework developed by Palaka et al. (1998), where a single product is offered by the firm, the system is modeled as a single $M/M/1$ queue, and the mean demand rate linearly decreases in price and guaranteed DT. So & Song (1998) investigated a quite similar problem while using a log-linear demand model (Cobb-Douglas). Different extensions have been investigated. Some studies consider the impact of the quoted DT on the costs. For instance, Ray & Jewkes (2004) incorporated the economies of scale by assuming that the unit operating cost is a convex decreasing function in the mean demand. Albana et al. (2018) modeled the unit operating cost as a convex decreasing function in the quoted DT. Webster (2002) assumed that the capacity (production rate) can change dynamically, which changes the variable production cost. Li et al. (2014) analyzed the benefit yielded from demand against the holding and tardiness penalty costs caused by the actual realized lead time being earlier or later than the promised DT.

Another extension consists in offering substitutable products differentiated in terms of DT

and price. For instance, Boyaci & Ray (2003) considered a firm that sells two substitutable products (a regular product with a standard DT, and an express product with a faster DT). Each product variety is served from a different facility. The study has been extended by Boyaci & Ray (2006) to incorporate the service level as a decision variable. Zhao et al. (2012) compared the strategy where a firm offers a single DT and price quotation to the strategy where a firm offers a menu of DTs and prices to customers. The substitutable products may also be offered by competing firms, where DT and price decisions for each firm are centralized. So (2000) studied DT- and price-based competition for service firms. Shang & Liu (2011) investigated the problem of two firms that offer substitutable products and compete in a DT-sensitive market, but did not consider pricing. Jayaswal & Jewkes (2016) considered a duopoly market with price or time sensitive customers (two segments) where the firms may have different operations strategy (shared versus dedicated capacity).

The DT quotation and pricing with inventory control has also been the focus of some interesting works. For instance, Yang & Geunz (2007) studied continuous-review inventory systems to maximize total profit when demand is convex in DT and price. Wu et al. (2012) considered the news-vendor problem with price- and DT-sensitive demand where a firm orders a semi-finished product prior to the selling season and customizes the final product in response to customer orders during the selling season. Savaşaneril & Sayin (2017) studied the relation between inventory level and DT quotes for a manufacturer serving multiple customer classes. Customers are sensitive to DT but pricing is not considered.

Dynamic DT quotation is another interesting extension. Savaşaneril et al. (2010) studied this problem in an $M/M/1$ base-stock inventory queue with DT-sensitive demand but without considering pricing. Feng et al. (2011) studied the joint dynamic price and DT quotation problem while modeling the system as an $GI/M/1$ queue. Hafızoğlu et al. (2016) considered two customer classes: contract customers whose orders are fulfilled based on a contract price and DT agreed on at the beginning of the time horizon, and spot purchasers who arrive over time and are quoted a pair of price and DT dynamically. Feng & Zhang (2017) investigated the dynamic DT quotation and pricing for an MTO manufacturer who can obtain information on individual sensitivity of each customer through negotiations with customers. Öner-Közen & Minner (2018) developed a Markov decision model to study the case of a firm who faces inhomogeneous customers and makes individual price and DT quotes to control its demand and decrease tardiness penalties. The authors compared the sequential decision process between

production and marketing to the simultaneous process. While the above studies consider a single operation stage, Hammami et al. (2020) studied the DT quotation for a single firm that has two operation stages under a centralized setting, but do not consider the upstream price (since only the final price is relevant in the centralized context).

The above studies consider a single firm with a centralized decision process in most cases. The trade-offs that govern the centralized problem are based on the overall SC performance while ignoring the profits of individual players. Consequently, how to apply the findings of these studies to analyze the decentralized problem where there are DT and price decisions for each firm is not clear.

2.2 Multiple firms and decentralized decision process

A first research stream investigates the dual-channel problem where a manufacturer can sell a product directly to the customers (direct channel) or through a retailer (indirect channel). The demand for each channel is typically linear with substitution and depends on manufacturer's DT. In Hua et al. (2010), the manufacturer (acting as a leader) decides the price and quoted DT for the direct channel, and the retailer (acting as a follower) decides its own price. Quoting a shorter DT incurs a higher delivery cost. Modak & Kelle (2019) extended this research by adding a stochastic exogenous demand (to the deterministic endogenous demand) and considering inventories at both manufacturer and retailer sites as additional decision variables. Xu et al. (2012) studied the delivery channel configuration where the manufacturer can use the direct, indirect, or both channels. Liu et al. (2019) studied a quite similar problem while assuming that the manufacturer provides a pay-on-delivery service directly to consumers through its online (direct) channel. The consumers' acceptance of a direct channel depends on the DT decided by the manufacturer. Most of these works consider a deterministic setting. Yang et al. (2017) proposed a news-vendor model to study the dual-channel SC where the switchover of customers is governed by the direct channel DT and the stockouts in both direct and retail channels. Pricing decisions were ignored. These works are different from our study as they consider a manufacturer-retailer SC where the retailer does not have a DT. In addition, the processing time is assumed to be deterministic in these works.

The stochastic models are more closely related to our work. We therefore review these studies in more details. Liu et al. (2007) studied a decentralized supplier-retailer SC where the supplier, as a leader, decides the quoted DT (to the customers) and the wholesale price (to the retailer).

The retailer, as a follower, decides the final price. Using the performance of the corresponding centralized system as a benchmark, the authors showed that the decentralized decisions are inefficient and lead to inferior performance due to the double marginalization effect. Pekgün et al. (2008) studied the centralization and decentralization of pricing and DT decisions for an MTO firm facing a linear price- and DT-sensitive demand with either marketing or production as the leader. The production department quotes a DT, and the marketing department quotes a price. The authors observed, for instance, that a higher capacity results in greater flexibility and higher profit for a centralized firm, but does not necessarily result in a higher profit for a decentralized firm. This research has been extended by Pekgün et al. (2017) while considering two firms (with either marketing or production in the lead for each firm) that compete on price and DT decisions in a common market. Under intense price competition, the authors found that the firms may suffer from a decentralized structure. However, under intense DT competition, a decentralized strategy with marketing in the lead can result in significantly higher profits. Zhu (2015) considered a similar SC configuration while assuming that the supplier (as a leader) determines the capacity and wholesale price, and the retailer (as a follower) decides the final price and DT. The author demonstrated that the integration of capacity decisions could significantly reduce the loss caused by the double marginalization. Xiao & Qi (2016) considered a supplier who operates in make-to-stock (MTS) where stockout could not happen and an MTO manufacturer. In the basic model, the supplier chooses the wholesale price, and the manufacturer determines the resale price and the quoted DT. The authors also studied the case where the manufacturer determines the resale price, the quoted DT, and the delivery reliability or capacity. They investigated the coordination of the channel via an all-unit quantity discount contract under different scenarios.

We provide in Table 1 a classification of multi-firm (decentralized) models and compare them to our modeling framework. As shown in Table 1, the papers that studied a SC with two independent firms assumed that the DT quoted to the final customers is equal to the DT quoted by only one firm (upstream or downstream), whereas the other firm does not have operations and, therefore, its lead time equals zero. This framework may be justified in case of a supplier-retailer SC (which is the most widely studied case in the literature), but does not fit with the case of a SC with two manufacturing firms (e.g., a manufacturer of semi-finished product and a manufacturer of final product). We consider a more general framework where each firm has a finite capacity, performs operations and has a lead time. Our demand is function of both

upstream and downstream DTs in addition to price. To our knowledge, this is the first study to consider this framework. Furthermore, while most published papers assume the DT to be controlled by the upstream firm (except Xiao & Qi 2016 and Zhu 2015), we study different situations where the DTs can be controlled by either the upstream or the downstream.

Table 1. Classification of relevant papers on DT quotation under a decentralized setting

	Capacitated firms	Decision variables	Lead times controlled by	Pricing controlled by	Demand function	Modeling framework
Liu et al. (2007)	1	l_u, p_u p_d	Upstream	Upstream and Downstream	$f(l_u, p_d)$	$M/M/1$ queue
Pekgün et al. (2008)	1	l_u, p_d	Upstream (production)	Downstream (marketing)	$f(l_u, p_d)$	$M/M/1$ queue
Hual et al. (2010)	1	l_u, p_u p_d	Upstream	Upstream and Downstream	$f(l_u, p_u, p_d)$	Dual-channel (Deterministic setting)
Xu et al. (2012)	1	l_u, p_u p_d	Upstream	Upstream and Downstream	$f(l_u, p_u, p_d)$	Dual-channel (Deterministic setting)
Zhu (2015)	1	l_u, p_u p_d, C_u	Downstream	Upstream and Downstream	$f(l_u, p_d)$	$M/M/1$ queue
Xiao & Qi (2016)	1	l_d, p_u p_d, s	Downstream	Upstream and Downstream	$f(l_d, p_d, s)$	$M/M/1$ queue
Pekgün et al. (2017)	1	l_u, p_d	Upstream (production)	Downstream (marketing)	$f(l_u, p_d)$	Competition 2 $M/M/1$ in parallel
Yang et al. (2017)	1	l_u, q_u q_d	Upstream	No pricing	$f(l_u, q_u, q_d)$	Dual-channel (Deterministic DT)
Modak & Kelle (2019)	1	l_u, p_u, p_d q_u, q_d	Upstream	Upstream and Downstream	$f(l_u, p_u, p_d)$	Dual-channel (Deterministic DT)
Liu et al. (2019)	1	l_u, p_u p_d	Upstream	Upstream and Downstream	$f(l_u, p_u, p_d)$	Dual-channel (Deterministic setting)
Our paper	2	l_u, l_d p_u, p_d	Upstream and/ or Downstream	Upstream and/ or Downstream	$f(l_u, l_d, p_d)$	Tandem queue $M/M/1 - M/M/1$

u : subscript denoting upstream firm, d : subscript denoting downstream firm, $f(\cdot)$: function of.

l : DT, p : price, q : order quantity (inventory), C : capacity, s : service level.

3 Modeling framework

We consider a two-stage SC consisting of a supplier (henceforth referred to as upstream firm) that represents a first manufacturing company and a buyer (henceforth referred to as downstream firm) that represents a second manufacturing company or a retailer. The downstream buys the upstream product and uses it to obtain the final product that is sold to the customers. There is an independent decision-maker for each stage. Both firms operate in an MTO setting and have a finite capacity. We assume that the service (processing) time in each stage is exponentially distributed. This assumption, widely used in the related literature (as shown in our literature review), is reasonable in case of high processing time variability, which is common in practice. We let μ_1 and μ_2 denote the mean service rate in upstream and downstream firms, respectively. As usually assumed in the related literature, the demand arrives at the downstream stage according to a Poisson process with mean arrival rate λ that decreases in the selling price and the overall DT quoted to customers. Thus, each firm is modeled as an $M/M/1$ queue, and the whole SC is represented by a tandem queue $M/M/1 - M/M/1$. We respectively denote by w_1 and w_2 the sojourn times in upstream and downstream firms. In a tandem queue $M/M/1 - M/M/1$, the sojourn time in each stage is exponentially distributed with mean $\frac{1}{\mu_1 - \lambda}$ and $\frac{1}{\mu_2 - \lambda}$, respectively. The sojourn time in the whole SC is given by $w_1 + w_2$.

We denote by l_1 and l_2 the local DTs in the upstream and downstream firms, respectively. The DT quoted to the customers is the overall DT of the SC, which is given by $l = l_1 + l_2$. Both firms know the minimum service level s that must be guaranteed by the whole SC, and each firm tries to satisfy this service level locally. Thus, the probability of satisfying the local DT in each stage must not be smaller than s , i.e., $\Pr(w_1 \leq l_1) \geq s$ and $\Pr(w_2 \leq l_2) \geq s$. In this case, if $s \geq 0.715$ (which is the common situation in practice), then the whole SC can quote the overall DT $l = l_1 + l_2$ to the customers while guaranteeing the minimum service level s (i.e., $\Pr(w_1 + w_2 \leq l) \geq s$). The proof is out of the scope of this paper; it can be found in (Albana 2018). In our model, both stages target the same service level s locally in order to guarantee that the whole SC can quote the DT l to customers while satisfying s . This implicitly assumes that there is no negotiation and coordination between the firms in the sense that one of them accepts to set a higher service level in order to allow the other company to target a lower service level while satisfying the service level s globally. This assumption is acceptable for two main reasons. First, for many companies, the service levels are viewed as constraints to be satisfied

and not as decision variables to be optimized. It is therefore possible that companies target the same service level, especially when they are independent (as considered in our problem) and operate in the same sector (such as in clinical diagnostics and industrial microbiology, a sector where many companies target a service level of 97%, as highlighted by our industrial partner Biomérieux). Second, increasing (respectively, decreasing) the service level of one stage means that we impose a tougher (respectively, weaker) constraint for that stage. This can be captured in our models by varying the capacity μ_i since we consider that each firm may have a different capacity.

We let p_1 denote the selling price of upstream product (i.e., purchasing price for the downstream), and p_2 denotes the downstream price (i.e., price of the final product sold to customers). For ease of presentation, we use p and p_2 alternatively to refer to the downstream price. We model the mean demand rate λ as a linearly decreasing function in the guaranteed DT l and price p of the final product. Thus, $\lambda = a - \alpha p - \beta l = a - \alpha p_2 - \beta(l_1 + l_2)$, where a is the market potential, α is the price-sensitivity of demand, and β is the DT-sensitivity. It is recalled that most related works assume linear demand functions. The unit margin for upstream and downstream firms is denoted by δ_1 and δ_2 , respectively, and $\delta = \delta_1 + \delta_2$. We denote by m_1 and m_2 the operating (production) cost per unit of product in upstream and downstream firms, respectively, and $m = m_1 + m_2$. Therefore, $\delta_1 = p_1 - m_1$ and $\delta_2 = p_2 - p_1 - m_2$. The studied system is illustrated in Figure 1.

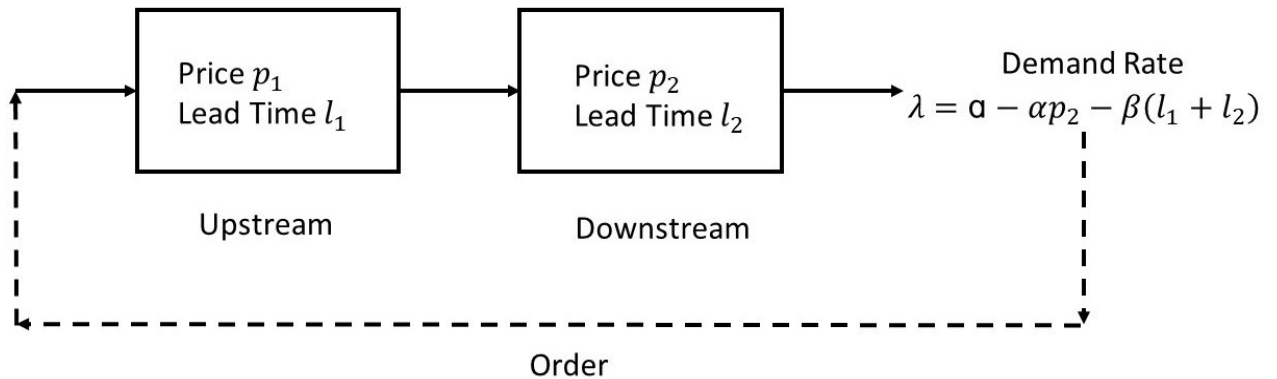


Figure 1. Modeling framework

Each firm undertakes DT and/or price decisions to maximize its local profit. We assume that each firm has a full information about the capacity of the other stage. As explained earlier, three decision settings will be studied in this paper. In Sections 4, 5 and 6, we respectively motivate each setting and develop the corresponding model and the analytical analysis.

4 Downstream controls lead times and each firm decides its price

In this first setting, the downstream, acting as a leader, decides its price p_2 and controls both DTs l_1 and l_2 , and the upstream, acting as a follower, reacts by deciding its own price p_1 . This decision scenario is motivated by practical situations in SCs where the downstream firm (manufacturer or assembler) has much more power than the upstream firm (supplier of a component or a semi-finished product) and can, therefore, control the upstream DT. Kraiselburd et al. (2010) state that *"in some SC relationships, there are downstream parties with the power to impose lead times reduction on their suppliers. For example, in automotive SCs, manufacturers have, to a certain extent, the ability to enforce short supply lead times"*. Magnusson & Simonsson (2012) reports that Mölnlycke Health Care (MHC), a world leading manufacturer of single-use surgical and wound care products adopted a new procurement strategy within the Surgical division stating that no suppliers should have a lead time that exceeds a certain number of weeks, which clearly means that MHC imposes DTs to the suppliers. Other practical examples can be found in the retail industry. Bose & Layne (2016) explain how some big retailers are getting tough with vendors to speed up the SC. The authors report that *"Target, the sixth-largest U.S. retailer by sales plans to tighten deadlines for deliveries to its warehouses"* and *"Wal-Mart is also cutting the window for deliveries to within one to two days of a target date, depending on the product category, from one to four days previously"*. The case where the downstream controls the DT of the upstream firm has been considered by Zhu (2015) in the retail context, but the authors do not consider the downstream DT (see Table 1).

The formulation of both upstream and downstream models is provided below. Note that $p_2 = p_1 + m_2 + \delta_2$. Hence, choosing p_2 for the downstream is equivalent to choosing δ_2 . The objective functions (1) and (5) respectively maximize the expected local profit for upstream and downstream firms. Equations (2) and (6) give the mean demand rate λ . Each firm must satisfy the service level s , implying that $\Pr(w_1 \leq l_1) \geq s$ and $\Pr(w_2 \leq l_2) \geq s$ in upstream and downstream models, respectively. These service constraints are respectively expressed by constraints (3) and (7) since the sojourn time in an $M/M/1$ queue is exponentially distributed. It is noted that the service constraint at each firm depends not only on the local DT of this stage but also the DT of the other stage (since λ depends on both l_1 and l_2). This situation makes the solving approach more complicated. Finally, constraints (4) and (8), respectively, guarantee a

positive demand in upstream and downstream models. Notice that the steady-state conditions in each stage (i.e., $\lambda < \mu_1$ and $\lambda < \mu_2$ for upstream and downstream stages, respectively) are automatically satisfied if the service constraints (i.e., constraints (3) and (7), respectively) are satisfied. Since the service constraints are included in the model, we can remove the steady-state conditions without any impact on the model formulation and optimization (see Palaka et al. 1998, Pekgün et al. 2008, Albana et al. 2018).

Upstream model:

$$(M1.1) \text{ Maximize } \Pi_1(p_1) = (p_1 - m_1)\lambda \quad (1)$$

$$\text{Subject to } \lambda = a - \alpha(p_1 + m_2 + \delta_2) - \beta(l_1 + l_2) \quad (2)$$

$$1 - e^{-(\mu_1 - \lambda)l_1} \geq s \quad (3)$$

$$\lambda \geq 0 \quad (4)$$

Downstream model:

$$(M1.2) \text{ Maximize } \Pi_2(\delta_2, l_1, l_2) = \delta_2\lambda \quad (5)$$

$$\text{Subject to } \lambda = a - \alpha(p_1 + m_2 + \delta_2) - \beta(l_1 + l_2) \quad (6)$$

$$1 - e^{-(\mu_2 - \lambda)l_2} \geq s \quad (7)$$

$$\lambda \geq 0 \quad (8)$$

We solve the problem backward. It is first noted that the service constraint (3) is not necessarily binding. The upstream may have an interest in setting a higher price than the one obtained when $1 - e^{-(\mu_1 - \lambda)l_1} = s$, which implies a non-binding situation (i.e., $1 - e^{-(\mu_1 - \lambda)l_1} > s$ at optimality). These binding and non-binding cases require different analytical analyses. In Lemma 1, we characterize the binding condition for the upstream model and determine the optimal upstream price in function of downstream decisions. To simplify the notation, we let $z = \ln\left(\frac{1}{1-s}\right)$. All proofs are presented in the Appendix.

Lemma 1 *For given δ_2 , l_1 and l_2 , the optimal upstream price is $p_1^* = \max(p_1^B, p_1^{NB})$, where $p_1^B = \frac{z - \mu_1 + a - \alpha(\delta_2 + m_2) - \beta(l_1 + l_2)}{\alpha}$ is the optimal price in binding situation, and*

- $p_1^{NB} = \frac{a - \alpha(\delta_2 + m_2 - m_1) - \beta(l_1 + l_2)}{2\alpha}$ is the optimal price in non-binding situation.

The service constraint in the upstream model is binding (i.e., $p_1^B \geq p_1^{NB}$) if, and only if, the DTs l_1 and l_2 and the margin δ_2 decided by the downstream verify

$$\beta l_1^2 - (a - \alpha(\delta_2 + m) - 2\mu_1 - \beta l_2) l_1 - 2z \leq 0 \quad (9)$$

After obtaining the expression of p_1^* in function of downstream decisions (δ_2 , l_1 and l_2), we can now consider the downstream model. We first focus on the service constraint and derive the following result.

Lemma 2 *Whether or not the service constraint in the upstream model is binding, the service constraint in the downstream model is always binding, implying that $\lambda = \mu_2 - \frac{z}{l_2} = a - \alpha(p_1 + m_2 + \delta_2) - \beta(l_1 + l_2)$ at optimality.*

In each of the binding and non-binding situations for the upstream model, we can now use the results of Lemma 1 and 2 to obtain the expression of the optimal DT l_2 and optimal price p_1 for given δ_2 and l_1 . The result is presented in Lemma 3.

Lemma 3 *If service constraint in the upstream model is binding, then we have at optimality (for given δ_2 and l_1):*

$$l_2 = \frac{z}{\frac{z}{l_1} - \mu_1 + \mu_2}, \quad p_1 = \frac{\frac{z}{l_1} - \mu_1 + a - \alpha(\delta_2 + m_2) - \beta \left(l_1 + \frac{z}{\frac{z}{l_1} - \mu_1 + \mu_2} \right)}{\alpha}, \quad \text{and } \lambda = \mu_1 - \frac{z}{l_1}.$$

If service constraint in the upstream model is not binding, then we have at optimality (for given δ_2 and l_1):

$$\begin{aligned} l_2 &= \frac{a - \alpha(\delta_2 + m) - \beta l_1 - 2\mu_2 + \sqrt{(a - \alpha(\delta_2 + m) - \beta l_1 - 2\mu_2)^2 + 8\beta z}}{2\beta}, \\ p_1 &= \frac{a - \alpha(\delta_2 + m_2 - 3m_1) - \beta l_1 + 2\mu_2 - \sqrt{(a - \alpha(\delta_2 + m) - \beta l_1 - 2\mu_2)^2 + 8\beta z}}{4\alpha}, \quad \text{and} \\ \lambda &= \frac{a - \alpha(\delta_2 + m) - \beta l_1 + 2\mu_2 - \sqrt{(a - \alpha(\delta_2 + m) - \beta l_1 - 2\mu_2)^2 + 8\beta z}}{4}. \end{aligned}$$

Based on the above results, we replace p_1 and l_2 with their optimal values for given δ_2 and l_1 and deduce that the downstream has to solve the following model (note that we must have $l_1 \geq \frac{z}{\mu_1}$ since, otherwise, the upstream model cannot be feasible).

$$\text{Maximize}_{\delta_2 \geq 0, l_1 \geq \frac{z}{\mu_1}} \Pi_2(\delta_2, l_1) = \begin{cases} \Pi_2^B(\delta_2, l_1) = \delta_2 \left(\mu_1 - \frac{z}{l_1} \right), & \text{if } \delta_2 \text{ and } l_1 \text{ satisfy the binding condition in upstream model} \\ \Pi_2^{NB}(\delta_2, l_1) = \delta_2 \left(\frac{a - \alpha(\delta_2 + m) - \beta l_1 + 2\mu_2 - \sqrt{(a - \alpha(\delta_2 + m) - \beta l_1 - 2\mu_2)^2 + 8\beta z}}{4} \right), & \text{if } \delta_2 \text{ and } l_1 \text{ satisfy the non-binding condition in upstream model} \end{cases} \quad (10)$$

It is recalled that the binding condition in the upstream model is given in (9) (see Lemma 1). We replace l_2 with its expression in the binding situation (given in Lemma 3) and deduce by standard calculus that the service constraint in the upstream model is binding if, and only if, $\mu_1 - \frac{z}{l_1} \leq \frac{a - \alpha(\delta_2 + m) - \beta l_1 + 2\mu_2 - \sqrt{(a - \alpha(\delta_2 + m) - \beta l_1 - 2\mu_2)^2 + 8\beta z}}{4}$. This condition is equivalent to $\Pi_2^B(\delta_2, l_1) \leq \Pi_2^{NB}(\delta_2, l_1)$. For given δ_2 , it can easily be verified that $\Pi_2^B(l_1)$ is increasing in l_1 , whereas $\Pi_2^{NB}(l_1)$ is decreasing. In addition, $\Pi_2^B(\frac{z}{\mu_1}) \leq \Pi_2^{NB}(\frac{z}{\mu_1})$, and $\lim_{l_1 \rightarrow +\infty} \Pi_2^B(l_1) \geq \lim_{l_1 \rightarrow +\infty} \Pi_2^{NB}(l_1)$. Hence, there is a unique value of l_1 such that $\Pi_2^B(l_1) = \Pi_2^{NB}(l_1)$, and we have $\Pi_2^{NB}(l_1) > \Pi_2^B(l_1)$ before this intersection point and $\Pi_2^{NB}(l_1) < \Pi_2^B(l_1)$ after it. This is illustrated in Figure 2 given in the appendix.

Consequently, for given δ_2 , the downstream profit $\Pi_2(l_1)$ is given by $\Pi_2^B(l_1)$ before the intersection point and by $\Pi_2^{NB}(l_1)$ after the intersection point. Moreover, given that $\Pi_2^B(l_1)$ is increasing in l_1 and $\Pi_2^{NB}(l_1)$ is decreasing, the optimal DT l_1^* (for given δ_2) corresponds to the intersection point between the curve of $\Pi_2^B(l_1)$ and $\Pi_2^{NB}(l_1)$. Therefore, we finally deduce that we have $\Pi_2^B(l_1) = \Pi_2^{NB}(l_1)$ at optimality, which is equivalent to $\mu_1 - \frac{z}{l_1} = \frac{a - \alpha(\delta_2 + m) - \beta l_1 + 2\mu_2 - \sqrt{(a - \alpha(\delta_2 + m) - \beta l_1 - 2\mu_2)^2 + 8\beta z}}{4}$. We show by standard calculus that this equation is equivalent to the following one.

$$\delta_2 = \frac{1}{\alpha} \left(a - \beta l_1 - \frac{\beta z}{\left(\mu_2 - \mu_1 + \frac{z}{l_1} \right)} - 2 \left(\mu_1 - \frac{z}{l_1} \right) \right) - m \quad (11)$$

The above equation gives δ_2 in function of l_1 at optimality. Thus, the downstream model is equivalent to the following one.

$$\text{Maximize}_{l_1 \geq \frac{z}{\mu_1}} \Pi_2(l_1) = \left(\mu_1 - \frac{z}{l_1} \right) \left[\frac{1}{\alpha} \left(a - \beta l_1 - \frac{\beta z}{\left(\mu_2 - \mu_1 + \frac{z}{l_1} \right)} - 2 \left(\mu_1 - \frac{z}{l_1} \right) \right) - m \right] \quad (12)$$

Notice that we can also replace the different variables with their expressions in function of

l_1 and deduce the upstream profit Π_1 and the overall profit $\Pi_G = \Pi_1 + \Pi_2$ as given below.

$$\Pi_1(l_1) = \frac{1}{\alpha} \left(\mu_1 - \frac{z}{l_1} \right)^2 \quad (13)$$

$$\Pi_G(l_1) = \frac{1}{\alpha} \left(\mu_1 - \frac{z}{l_1} \right) \left[a - \beta l_1 - \frac{\beta z}{\left(\mu_2 - \mu_1 + \frac{z}{l_1} \right)} - \left(\mu_1 - \frac{z}{l_1} \right) - \alpha m \right] \quad (14)$$

Given the complexity of the expression of $\Pi_2(l_1)$, it is not possible to obtain the closed-form expression of l_1^* . We describe the optimal strategy in the following proposition.

Proposition 1 *Optimal solution under setting 1.*

- Optimal delivery time l_1^* is obtained by maximizing

$$\left(\mu_1 - \frac{z}{l_1} \right) \left[\frac{1}{\alpha} \left(a - \beta l_1 - \frac{\beta z}{\left(\mu_2 - \mu_1 + \frac{z}{l_1} \right)} - 2 \left(\mu_1 - \frac{z}{l_1} \right) \right) - m \right] \text{ over } \left[\frac{z}{\mu_1}, +\infty \right[.$$

- Optimal delivery time $l_2^* = \frac{z l_1^*}{z + (\mu_2 - \mu_1) l_1^*}$.

- Optimal upstream margin $\delta_1^* = \frac{1}{\alpha} \left(\mu_1 - \frac{z}{l_1^*} \right)$, and optimal upstream price $p_1^* = \frac{1}{\alpha} \left(\mu_1 - \frac{z}{l_1^*} \right) + m_1$.

- Optimal downstream margin $\delta_2^* = \frac{1}{\alpha} \left[a - \beta l_1^* - \frac{\beta z}{\left(\mu_2 - \mu_1 + \frac{z}{l_1^*} \right)} - 2 \left(\mu_1 - \frac{z}{l_1^*} \right) \right] - m$, and optimal downstream price $p_2^* = \frac{1}{\alpha} \left[a - \beta l_1^* - \frac{\beta z}{\left(\mu_2 - \mu_1 + \frac{z}{l_1^*} \right)} - \left(\mu_1 - \frac{z}{l_1^*} \right) \right]$.

5 Downstream controls prices and each firm decides its lead time

Bertini & Koenigsberg (2014) identify three general philosophies to pricing: company-imposed pricing, collaborative pricing and customer-imposed pricing. According to the authors, under customer-imposed pricing, *"the company delegates responsibility for pricing to the customer"*. In B2B, customers may have aggressive purchasing strategies which finally lead to imposing prices to suppliers. Examples can be found in the automotive industry where some powerful automakers impose price reductions to their suppliers on a regular basis. One of the authors of this paper, who has been working as a purchasing manager in the automotive sector, reports the example of a manufacturer of automotive components who outsources the production of some

plastic parts. The manufacturer provides its supplier with the mold, but then imposes the price of the product to the supplier. In this second setting, we consider the case of customer-imposed pricing. Thus, the downstream, acting, as a leader, decides its DT l_2 and controls prices p_1 and p_2 . The upstream reacts by choosing its own DT l_1 . The obvious limitation of this pricing strategy is that the downstream may be tempted to undermine the upstream with unreasonably low prices, which will not be acceptable. One way companies can mitigate this risk is to set a price floor (Bertini & Koenigsberg 2014). Hence, to prevent the downstream from imposing unreasonable prices, we consider here that the unitary upstream margin (i.e., $p_1 - m_1$) cannot be smaller than a minimum value denoted by δ_1^{\min} . Thus, although the downstream firm decides the prices, it must respect the minimum price required by the upstream. The margin floor δ_1^{\min} can be negotiated beforehand between both parties. The value of δ_1^{\min} may reflect the power of each firm in the SC. As δ_1^{\min} would influence the optimal strategies and profits, we will conduct a sensitivity analysis to understand its effect.

Since $p_2 = p_1 + m_2 + \delta_2$, the constraint $p_1 - m_1 \geq \delta_1^{\min}$ is equivalent to $p_2 \geq m + \delta_2 + \delta_1^{\min}$. The formulation of the upstream and downstream models is given below. We recall that the steady-state conditions can be removed from the analysis as explained earlier. To simplify the presentation and analysis of the problem, the downstream model (M2.2) provided here is an equivalent formulation of the original model, obtained by replacing p_2 with $p_1 + m_2 + \delta_2$.

Upstream model:

$$(M2.1) \text{ Maximize } \Pi_1(l_1) = (p_1 - m_1)\lambda \quad (15)$$

$$l_1 \geq 0,$$

$$\text{Subject to } \lambda = a - \alpha(p_1 + m_2 + \delta_2) - \beta(l_1 + l_2) \quad (16)$$

$$1 - e^{-(\mu_1 - \lambda)l_1} \geq s \quad (17)$$

$$\lambda \geq 0 \quad (18)$$

Downstream model:

$$(M2.2) \quad \underset{\delta_2 \geq 0, l_2 \geq 0, p_1 \geq m_1 + \delta_1^{\min}}{\text{Maximize}} \quad \Pi_2(l_2, \delta_2, p_1) = \delta_2 \lambda \quad (19)$$

$$\text{Subject to} \quad \lambda = a - \alpha(p_1 + m_2 + \delta_2) - \beta(l_1 + l_2) \quad (20)$$

$$1 - e^{-(\mu_2 - \lambda)l_2} \geq s \quad (21)$$

$$\lambda \geq 0 \quad (22)$$

We solve the problem by backward induction. We first assume that l_2, δ_2 and p_1 are given and solve the upstream model to obtain l_1 in function of l_2, δ_2 and p_1 at optimality. We then replace l_1 in the downstream model with its optimal expression and solve the latter model.

Upstream model (M2.1).

Service constraint (17) is equivalent to $l_1 \geq \frac{z}{\mu_1 - \lambda}$. We replace λ with its expression given in equation (16) and deduce that constraint (17) is equivalent to $\Phi(l_1) \geq 0$, where $\Phi(l_1) = \beta l_1^2 - (a - \alpha(p_1 + m_2 + \delta_2) - \beta l_2 - \mu_1) l_1 - z$. It can be verified that the quadratic equation $\Phi(l_1) = 0$ has only one positive root, $l_1^{\min} = \frac{a - \alpha(p_1 + m_2 + \delta_2) - \beta l_2 - \mu_1 + \sqrt{(a - \alpha(p_1 + m_2 + \delta_2) - \beta l_2 - \mu_1)^2 + 4\beta z}}{2\beta}$, and that $\Phi(l_1) \geq 0 \Leftrightarrow l_1 \geq l_1^{\min}$. Moreover, $\lambda \geq 0 \Leftrightarrow l_1 \leq l_1^{\max} = \frac{a - \alpha(p_1 + m_2 + \delta_2) - \beta l_2}{\beta}$. Hence, for given l_2, δ_2 , and p_1 , model (M2.1) is equivalent to the following model.

$$\underset{l_1}{\text{Maximize}} \quad \Pi_1(l_1) = (p_1 - m_1) (a - \alpha(p_1 + m_2 + \delta_2) - \beta(l_1 + l_2)) \quad (23)$$

$$\text{Subject to} \quad l_1^{\min} \leq l_1 \leq l_1^{\max} \quad (24)$$

It is clear that $\Pi_1(l_1)$ is decreasing in l_1 . Therefore, the upstream will choose the shortest DT while satisfying the constraints, which implies that $l_1 = l_1^{\min}$ at optimality. Thus, the optimal upstream DT is given by equation (25). This also means that the service constraint in the upstream model is binding and $\lambda = \mu_1 - \frac{z}{l_1}$ at optimality.

$$l_1 = \frac{a - \alpha(p_1 + m_2 + \delta_2) - \beta l_2 - \mu_1 + \sqrt{(a - \alpha(p_1 + m_2 + \delta_2) - \beta l_2 - \mu_1)^2 + 4\beta z}}{2\beta} \quad (25)$$

Furthermore, we need to ensure that downstream's decisions must also satisfy $l_1^{\min} \leq l_1^{\max}$ since, otherwise, the upstream model is not feasible (i.e., it is not possible to quote a DT

that simultaneously satisfies the service constraint and guarantees a positive demand). This condition is equivalent to constraint (26) given below. This constraint must be added to the downstream model.

$$l_2 \leq \frac{a - \alpha(p_1 + m_2 + \delta_2)}{\beta} - \frac{z}{\mu_1} \quad (26)$$

Downstream model (M2.2).

We now focus on the downstream model. Model (M2.2) is challenging since λ depends not only on l_2, δ_2 and p_1 but also l_1 , which has been expressed in function of l_2, δ_2 and p_1 in equation (25). To simplify the formulation of model (M2.2), we first derive the following Lemma.

Lemma 4 *Service constraint in the downstream model is binding (i.e., $\lambda = \mu_2 - \frac{z}{l_2}$ at optimality).*

According to Lemma 4, we have $\lambda = \mu_2 - \frac{z}{l_2}$ at optimality. Therefore, we can replace λ with $\mu_2 - \frac{z}{l_2}$. Equation (20) becomes equivalent to $\mu_2 - \frac{z}{l_2} = a - \alpha(p_1 + m_2 + \delta_2) - \beta(l_1 + l_2)$. In this equation, we replace l_1 with its expression at optimality and obtain by standard calculus the price p_1 in function of δ_2 and l_2 .

$$p_1 = \frac{1}{\alpha} \left(a - \alpha(\delta_2 + m_2) - \beta l_2 - \mu_2 + \frac{z}{l_2} - \frac{\beta z}{\frac{z}{l_2} - \mu_2 + \mu_1} \right) \quad (27)$$

Remark 1 *If $\mu_2 > \mu_1$, then p_1 is not continuous at the point $l_2 = \frac{z}{\mu_2 - \mu_1}$ (for given δ_2), as illustrated in Figure 3 (see the appendix). However, this discontinuity does not impact our analysis since only the region $\left[\frac{z}{\mu_2}, \frac{z}{\mu_2 - \mu_1} \right]$ is relevant to our study (see the appendix).*

We now use the above results to provide an equivalent formulation of model (M2.2). It is noted that $\lambda \geq 0$ is equivalent to $l_2 \geq \frac{z}{\mu_2}$ and constraint (26) must be added to guarantee the feasibility of the upstream model. Hence, model (M2.2) is equivalent to the following model.

$$\underset{l_2 \geq 0, \delta_2 \geq 0, p_1 \geq m_1 + \delta_1^{\min}}{\text{Maximize}} \quad \Pi_2(l_2, \delta_2, p_1) = \delta_2 \left(\mu_2 - \frac{z}{l_2} \right) \quad (28)$$

$$\text{Subject to } p_1 = \frac{1}{\alpha} \left(a - \alpha(\delta_2 + m_2) - \beta l_2 - \mu_2 + \frac{z}{l_2} - \frac{\beta z}{\frac{z}{l_2} - \mu_2 + \mu_1} \right) \quad (29)$$

$$l_2 \leq \frac{a - \alpha(p_1 + m_2 + \delta_2)}{\beta} - \frac{z}{\mu_1} \quad (30)$$

$$l_2 \in \left[\frac{z}{\mu_2}, \frac{z}{\mu_2 - \mu_1} \right] \text{ if } \mu_2 > \mu_1 \text{ and } l_2 \in \left[\frac{z}{\mu_2}, +\infty \right] \text{ if } \mu_2 \leq \mu_1 \quad (31)$$

For given δ_2 , Π_2 is clearly increasing in l_2 . Therefore, the downstream will try to increase l_2 . However, this act will lead to decreasing p_1 (since $\frac{\partial p_1(\delta_2, l_2)}{\partial l_2} = \frac{-1}{\alpha} \left(\beta + \frac{z}{l_2^2} + \frac{\beta z^2}{(z - (\mu_2 - \mu_1) l_2) l_2} \right) < 0$), which cannot be smaller than $m_1 + \delta_1^{\min}$. When p_1 is at its lowest possible value (i.e., $p_1 = m_1 + \delta_1^{\min}$), we deduce from constraint (30) that l_2 must be smaller than $l_2^{\max} = \frac{a - \alpha(m + \delta_1^{\min} + \delta_2)}{\beta} - \frac{z}{\mu_1}$ (we recall that $m = m_1 + m_2$). Thus, if p_1 reaches $m_1 + \delta_1^{\min}$ before l_2 reaches l_2^{\max} (which is mathematically equivalent to $p_1(\delta_2, l_2^{\max}) \leq m_1 + \delta_1^{\min}$), then $p_1^* = m_1 + \delta_1^{\min}$ since, on the one hand, this solution is feasible and, on the other hand, it is not possible to increase l_2 again. One remark is in order here.

Remark 2 *The case $p_1(\delta_2, l_2^{\max}) > m_1 + \delta_1^{\min}$ is not relevant to our study as it cannot generate profit (see Appendix).*

We, therefore, focus on the relevant case; that is, when $p_1(\delta_2, l_2^{\max}) \leq m_1 + \delta_1^{\min}$. In this case, we have $p_1^* = m_1 + \delta_1^{\min}$. In other words, the minimum margin required by the upstream will be its effective margin at optimality. Hence, on the one hand, p_1 is given by equation (29) and, on the other hand, we have $p_1 = m_1 + \delta_1^{\min}$ at optimality. Therefore, we deduce δ_2 in function of l_2 at optimality as given in equation (32).

$$\delta_2 = \frac{(\mu_2 - \mu_1) \beta l_2^3 + ((\mu_2 - \mu_1)(\mu_2 - a + \alpha(m + \delta_1^{\min})) - 2\beta z) l_2^2 + (\mu_1 - 2\mu_2 + a - \alpha(m + \delta_1^{\min})) z l_2 + z^2}{\alpha(z - (\mu_2 - \mu_1) l_2) l_2} \quad (32)$$

We replace p_1 and δ_2 with their expressions at optimality and finally deduce that model (M2.2) is equivalent to the following single-variable optimization model.

$$\begin{aligned} \text{Maximize}_{l_2} \quad \Pi_2(l_2) &= \frac{\mu_2 - \frac{z}{l_2}}{\alpha(z - (\mu_2 - \mu_1)l_2)l_2} \left[\begin{aligned} &(\mu_2 - \mu_1)\beta l_2^3 + ((\mu_2 - \mu_1)(\mu_2 - a + \alpha(m + \delta_1^{\min})) - 2\beta z)l_2^2 \\ &+ (\mu_1 - 2\mu_2 + a - \alpha(m + \delta_1^{\min}))zl_2 + z^2 \end{aligned} \right] \\ \text{Subject to} \quad l_2 &\in \left[\frac{z}{\mu_2}, \frac{z}{\mu_2 - \mu_1} \right] \text{ if } \mu_2 > \mu_1 \text{ and } l_2 \in \left[\frac{z}{\mu_2}, +\infty \right] \text{ if } \mu_2 \leq \mu_1 \end{aligned} \quad (33)$$

Notice that we can also replace the different variables with their expressions in function of l_2 and deduce the upstream profit Π_1 and the overall profit of the SC $\Pi_G = \Pi_1 + \Pi_2$ as given below.

$$\Pi_1(l_2) = \delta_1^{\min} \left(\mu_2 - \frac{z}{l_2} \right) \quad (34)$$

$$\Pi_G(l_2) = \left(\mu_2 - \frac{z}{l_2} \right) \left[\begin{aligned} &\delta_1^{\min} + \\ &\frac{(\mu_2 - \mu_1)\beta l_2^3 + ((\mu_2 - \mu_1)(\mu_2 - a + \alpha(m + \delta_1^{\min})) - 2\beta z)l_2^2 + (\mu_1 - 2\mu_2 + a - \alpha(m + \delta_1^{\min}))zl_2 + z^2}{\alpha(z - (\mu_2 - \mu_1)l_2)l_2} \end{aligned} \right] \quad (35)$$

We finally provide the optimal strategy in the following proposition.

Proposition 2 *Optimal solution under setting 2.*

- *Optimal delivery time l_2^* is obtained by maximizing*

$$\left(\mu_2 - \frac{z}{l_2} \right) \frac{(\mu_2 - \mu_1)\beta l_2^3 + ((\mu_2 - \mu_1)(\mu_2 - a + \alpha(m + \delta_1^{\min})) - 2\beta z)l_2^2 + (\mu_1 - 2\mu_2 + a - \alpha(m + \delta_1^{\min}))zl_2 + z^2}{\alpha(z - (\mu_2 - \mu_1)l_2)l_2} \text{ over } \left[\frac{z}{\mu_2}, \frac{z}{\mu_2 - \mu_1} \right]$$

and $\left[\frac{z}{\mu_2}, +\infty \right]$ for $\mu_2 > \mu_1$ and $\mu_2 \leq \mu_1$, respectively.

- *Optimal delivery time $l_1^* = \frac{zl_2^*}{z - (\mu_2 - \mu_1)l_2^*}$.*

- *Optimal upstream margin $\delta_1^* = \delta_1^{\min}$, and optimal upstream price $p_1^* = m_1 + \delta_1^{\min}$.*

- *Optimal downstream margin*

$$\delta_2^* = \frac{(\mu_2 - \mu_1)\beta l_2^{*3} + ((\mu_2 - \mu_1)(\mu_2 - a + \alpha(m + \delta_1^{\min})) - 2\beta z)l_2^{*2} + (\mu_1 - 2\mu_2 + a - \alpha(m + \delta_1^{\min}))zl_2^* + z^2}{\alpha(z - (\mu_2 - \mu_1)l_2^*)l_2^*}, \text{ and optimal}$$

downstream price

$$p_2^* = m + \delta_1^{\min} + \frac{(\mu_2 - \mu_1)\beta l_2^{*3} + ((\mu_2 - \mu_1)(\mu_2 - a + \alpha(m + \delta_1^{\min})) - 2\beta z)l_2^{*2} + (\mu_1 - 2\mu_2 + a - \alpha(m + \delta_1^{\min}))zl_2^* + z^2}{\alpha(z - (\mu_2 - \mu_1)l_2^*)l_2^*}.$$

6 Upstream controls lead times and each firm decides its price

Some SCs are controlled by the upstream firm who may have more power than the downstream firm. Examples of powerful suppliers can be found in internet retailing (see Liu et al. 2007). Most related works on DT quotation with independent firms consider a supplier-retailer SC

dominated by the supplier and where the retailer just has a pricing role (see Table 1). When a big manufacturer has an exclusive distributor who just customizes the product and deliver it to the customers, the operational decisions, including lead times, may be controlled by the supplier. This section investigates the case where the upstream, acting as a leader, decides its price p_1 and control DTs l_1 and l_2 , and the downstream reacts by deciding its own price p_2 (which is equivalent to deciding the unit margin δ_2). The formulation of the models is given below. We recall that the steady-state conditions can be removed from the analysis as explained earlier.

Upstream model:

$$(M3.1) \text{ Maximize } \Pi_1(p_1, l_1, l_2) = (p_1 - m_1)\lambda \quad (36)$$

$$p_1 \geq 0, l_1 \geq 0, l_2 \geq 0$$

$$\text{Subject to } \lambda = a - \alpha(p_1 + m_2 + \delta_2) - \beta(l_1 + l_2) \quad (37)$$

$$1 - e^{-(\mu_1 - \lambda)l_1} \geq s \quad (38)$$

$$\lambda \geq 0 \quad (39)$$

Downstream model:

$$(M3.2) \text{ Maximize } \Pi_2(\delta_2) = \delta_2\lambda \quad (40)$$

$$\delta_2 \geq 0$$

$$\text{Subject to } \lambda = a - \alpha(p_1 + m_2 + \delta_2) - \beta(l_1 + l_2) \quad (41)$$

$$1 - e^{-(\mu_2 - \lambda)l_2} \geq s \quad (42)$$

$$\lambda \geq 0 \quad (43)$$

We use the same solving methodology as setting 1. Thus, to avoid redundancy, we provide the different solving steps in the appendix. We obtain the following equivalent formulation of upstream model in function of l_2 .

$$\text{Maximize } \Pi_1(l_2) = \left(\mu_2 - \frac{z}{l_2} \right) \left[\frac{1}{\alpha} \left(a - \beta l_2 - \frac{\beta z}{\left(\mu_1 - \mu_2 + \frac{z}{l_2} \right)} - 2 \left(\mu_2 - \frac{z}{l_2} \right) \right) - m \right] \quad (44)$$

$$l_2 \geq \frac{z}{\mu_2}$$

Notice that we can also replace the different variables with their expressions in function of l_2 and deduce the downstream profit Π_2 and the overall profit Π_G as given below.

$$\Pi_2(l_2) = \frac{1}{\alpha} \left(\mu_2 - \frac{z}{l_2} \right)^2 \quad (45)$$

$$\Pi_G(l_2) = \frac{1}{\alpha} \left(\mu_2 - \frac{z}{l_2} \right) \left[a - \beta l_2 - \frac{\beta z}{\mu_1 - \mu_2 + \frac{z}{l_2}} - \left(\mu_2 - \frac{z}{l_2} \right) - \alpha m \right] \quad (46)$$

We provide the optimal strategy in the following proposition.

Proposition 3 *Optimal solution under setting 3.*

- *Optimal delivery time l_2^* is obtained by maximizing*

$$\left(\mu_2 - \frac{z}{l_2} \right) \left[\frac{1}{\alpha} \left(a - \beta l_2 - \frac{\beta z}{\left(\mu_1 - \mu_2 + \frac{z}{l_2} \right)} - 2 \left(\mu_2 - \frac{z}{l_2} \right) \right) - m \right] \text{ over } \left[\frac{z}{\mu_2}, +\infty \right[.$$

- *Optimal delivery time $l_1^* = \frac{z l_2^*}{(\mu_1 - \mu_2) l_2^* + z}$.*

- *Optimal upstream margin $\delta_1^* = \frac{1}{\alpha} \left(a - \beta l_2^* - \frac{\beta z}{\left(\mu_1 - \mu_2 + \frac{z}{l_2^*} \right)} - 2 \left(\mu_2 - \frac{z}{l_2^*} \right) \right) - m$, and opti-*

$$\text{mal upstream price } p_1^* = \frac{1}{\alpha} \left(a - \beta l_2^* - \frac{\beta z}{\left(\mu_1 - \mu_2 + \frac{z}{l_2^*} \right)} - 2 \left(\mu_2 - \frac{z}{l_2^*} \right) \right) - m_2.$$

- *Optimal downstream margin $\delta_2^* = \frac{1}{\alpha} \left(\mu_2 - \frac{z}{l_2^*} \right)$, and optimal downstream price*

$$p_2^* = \frac{1}{\alpha} \left(a - \beta l_2^* - \frac{\beta z}{\left(\mu_1 - \mu_2 + \frac{z}{l_2^*} \right)} - \left(\mu_2 - \frac{z}{l_2^*} \right) \right).$$

7 Managerial insights

Based on the optimal solutions obtained in the previous sections, we now derive managerial insights into the interplay between local DTs, overall DT, prices, demand and profits. The main results are presented in a series of corollaries. All proofs are given in the appendix.

Corollary 1 *Under all settings, all DTs are positively correlated. Hence, the shorter the DT quoted by one firm, the shorter the DT quoted by the other firm and, consequently, the shorter the overall DT quoted by the SC.*

However, the upstream and the downstream prices are not always positively correlated.

It is first important to note that while local DTs (i.e., upstream and downstream DTs) are sensitive to market characteristics and production costs, the relation between these DTs depends only on service level and capacities (since we have in all settings $l_1 = \frac{zl_2}{(\mu_1 - \mu_2)l_2 + z}$ at optimality). In particular, both firms will quote the same DT ($l_1 = l_2$) if they have the same capacity. Corollary 1 shows that the local DTs are positively correlated whatever the decision scenario. Therefore, a firm will always decrease (respectively, increase) its DT if the other firm decreases (respectively, increases) its own DT. Practically, this means that the lead time efforts of both firms must always be done in the same direction. The case where one firm decreases its DT whereas the other firm increases its own DT should not occur in practice. This result also means that quoting a shorter (respectively, a longer) DT to final customers requires that both firms quote a shorter (respectively, a longer) DT locally.

Liu et al. (2007) and Xiao & Qi (2016) considered a two-stage decentralized SC where only one firm undertakes operations and has a DT (see Table 1). These works, which are closely related to our study, showed that the upstream and downstream prices are positively correlated (i.e., when one price increases, the other price also increases at optimality). Our study reveals that this result does not always hold if both firms have operations and DTs. In fact, Corollary 1 demonstrates that the relation between upstream and downstream prices depends on the decision scenario. Under setting 1, for instance, a longer DT leads the upstream price to increase and the downstream price to decrease. Under setting 2, the upstream price is constant at optimality while the downstream price is a complex function of DT (we can easily show that this function decreases in DT when $\mu_1 = \mu_2$). Under setting 3, both upstream and downstream prices decrease in quoted DT as found by Liu et al. (2007) and Xiao & Qi (2016). This means that the consideration of both DTs (instead of only one DT) changes the trade-offs that govern the pricing decisions. Therefore, it may be sub-optimal for firms to make pricing efforts in the same direction as shown in the literature. Our study shows that adopting opposite pricing strategies may be the optimal strategy. We now investigate the relation between DTs and demand.

Corollary 2 *Under all settings, the longer the quoted DT (in any stage of the SC), the larger the expected demand.*

The fact that quoting a longer DT leads to a higher demand is a surprising but known result in the literature with single operation stage (see e.g., Liu et al. 2007, Xiao & Qi 2016, Albana

et al. 2018). The reason is that a longer DT allows to serve more demand without violating the service constraint which leads to decreasing the price in order to increasing demand. We generalize this result by showing that it also holds when each firm has a DT. Since l is positively correlated with l_1 and l_2 (as demonstrated in Corollary 1), any local or global increase in DT implies a greater demand, but this does not necessarily generate a higher profit, as will be discussed in Corollary 3.

Corollary 3 *Under all settings, the follower’s expected profit increases in quoted DT (in any stage of the SC) and therefore increases in mean demand, whereas the SC leader’s expected profit is a concave function in quoted DT and in mean demand.*

To illustrate the above result, we consider the first setting and represent in Figures 4 and 5 the profits Π_1 , Π_2 , and the overall SC profit $\Pi_G = \Pi_1 + \Pi_2$ in function of l_1 in the cases of $\mu_1 = \mu_2 = 20$ and $\mu_1 = 15$ and $\mu_2 = 30$, respectively. We consider a basic numerical example with $a = 80$, $\alpha = 5$, $\beta = 5$, $m_1 = m_2 = 4$, and $s = 0.95$. Using the same numerical example, we also provide illustrations for settings 2 and 3 in Figures 6 and 7, respectively. We see in all cases how the follower’s expected profit increases in quoted DT while the SC leader’s expected profit is a concave function. Note that since all DTs are positively correlated, a profit that increases in one of them (e.g., l_1) also increases in the others (l_2 and l).

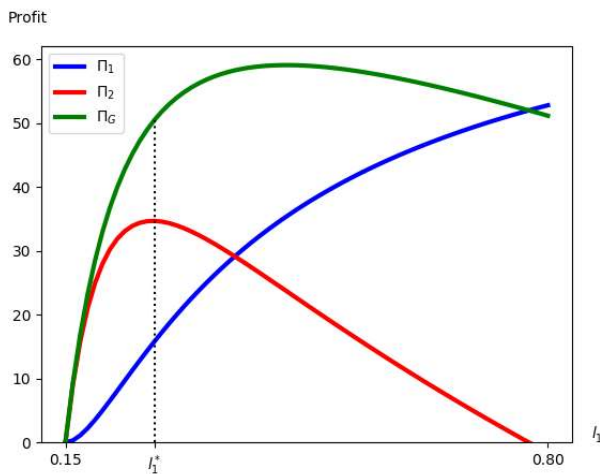


Figure 4. Profits vs. l_1 ($\mu_1 = \mu_2$) for setting 1

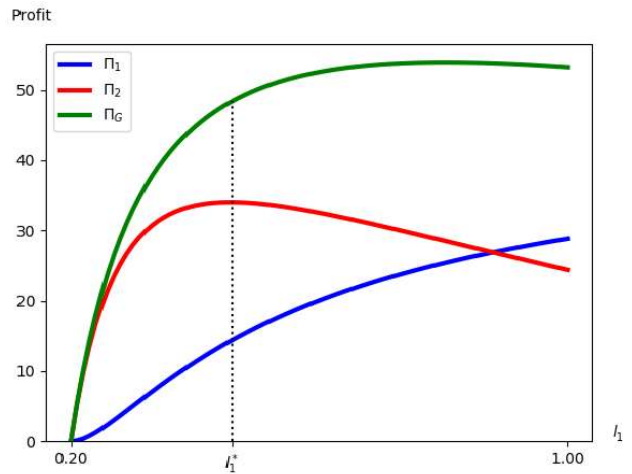


Figure 5. Profits vs. l_1 ($\mu_1 < \mu_2$) for setting 1

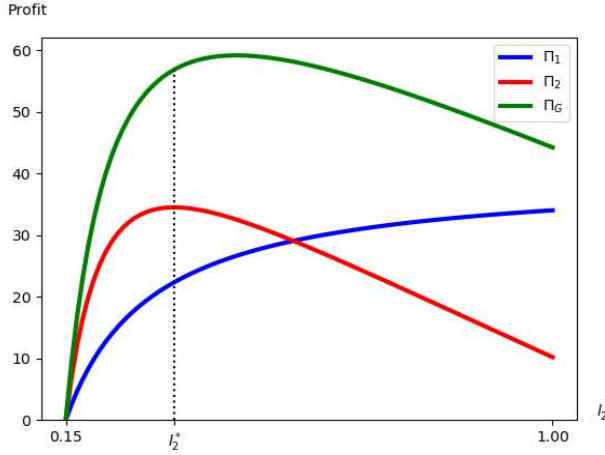


Figure 6. Profits vs. l_2 with $\delta_1^{\min} = 2$ for setting 2

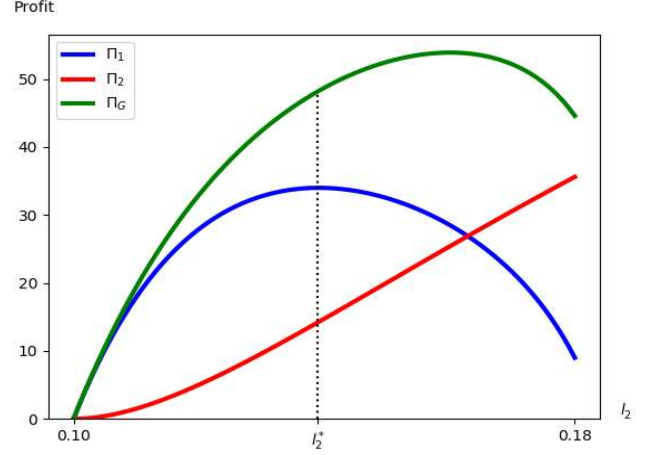


Figure 7. Profits vs. l_2 for setting 3

We first focus on setting 1. We see that l_1^* is the DT that maximizes $\Pi_2(l_1)$, but this DT does not maximize $\Pi_1(l_1)$ and $\Pi_G(l_1)$. The upstream would have preferred that the SC quotes a longer DT as its profit is increasing in l_1 , but l_1 is decided by the downstream, who aims only to maximizing its own profit. The upstream would have made more profit if the downstream requires a longer DT l_1 (than l_1^*) because this not only leads to a higher demand, but also allows the upstream to set a higher price p_1 . The decrease in final price p_2 (resulting from quoting a longer DT) is therefore due to the decrease in downstream margin δ_2 . This results in a downstream profit that is concave in DT. Thus, the downstream should first increase the DT to obtain a profitable demand although this incurs a lower margin but, after a threshold value, the DT becomes too long and the margin too small. Consequently, it becomes more profitable for the downstream to capitalize on increasing its margin rather than increasing demand. In summary, the upstream has interest in offering the final product to customers with the longest overall DT (l), but this incurs the smallest margin for the downstream, whose optimal strategy is a trade-off between: (1) increasing its margin by quoting a shorter DT, but this leads to a higher final price which incurs a lower demand, and (2) generating a higher demand by quoting a longer overall DT, but this leads to a smaller margin. As demand increases in DT (as shown in Corollary 2), the above analysis also implies that the upstream (follower) profit increases in demand while the downstream (leader) profit is a concave function in demand. This result is interesting as it differs from the findings of Liu et al. (2007) and Xiao & Qi (2016) where the profits of both firms increase in demand. Thus, while these studies suggest that increasing demand is profitable for both firms, our findings indicate that only the follower has always interest in increasing demand whereas the leader should find the right balance between demand

and price (since its profit decreases when demand becomes too large). As stated in the corollary and illustrated in Figures 6 and 7, this behavior is common to all our settings.

We now provide some specific results. First, we compare the outcomes of setting 1, in terms of final product characteristics and optimal profits, to the results obtained under setting 3. The main insight is given in the following corollary.

Corollary 4 *Consider the optimal strategy obtained under setting 1 when upstream and downstream stages have respectively the capacities μ_1 and μ_2 .*

Setting 3 leads to offering the final product to customers with the same DT and price as setting 1 if the upstream and downstream stages swap their capacities (i.e., if upstream and downstream stages have the capacities μ_2 and μ_1 , respectively). In this case (i.e., when capacities are swapped), the upstream and downstream optimal profits are also swapped and we obtain the same overall SC profit.

To illustrate, we compare the optimal solutions and profits of settings 1 and 3 for our basic numerical example with different capacities. The results are presented in Table 2.

Table 2. Setting 3 vs. Setting 1

		l_1^*	l_2^*	p_2^*	π_1^*	π_2^*	π_G^*
$\mu_1 = \mu_2 = 20$	<i>Setting 1</i>	0.27	0.27	13.68	15.83	34.71	50.54
$\mu_1 = \mu_2 = 20$	<i>Setting 3</i>	0.27	0.27	13.68	34.71	15.83	50.54
$\mu_1 = 10$ and $\mu_2 = 50$	<i>Setting 1</i>	0.88	0.07	13.72	8.75	29.11	37.85
$\mu_1 = 50$ and $\mu_2 = 10$	<i>Setting 3</i>	0.07	0.88	13.72	29.11	8.75	37.85
$\mu_1 = 50$ and $\mu_2 = 10$	<i>Setting 1</i>	0.07	1.05	13.45	10.23	28.73	38.96
$\mu_1 = 10$ and $\mu_2 = 50$	<i>Setting 3</i>	1.05	0.07	13.45	28.73	10.23	38.96

Most related papers study the case where only the firm that makes operations is the SC leader (see Table 1). We investigate cases where the leader can be either the downstream or the upstream and reveal an important finding that may be deduced from Corollary 4. Indeed, in case of balanced capacity (i.e., same capacity for both firms), the SC will make the same overall profit regardless of the firm who will decide the DTs and act as a leader. This result has also a practical interest for the final customers. It shows that the customers will be offered the product with the same price and DT regardless of the firm who will decide the DTs and act as a leader.

In setting 2, δ_1^{\min} plays an important role. We recall that δ_1^{\min} is the minimum margin required by the upstream and, therefore, represents the power of the upstream firm in the SC. The greater the value of δ_1^{\min} , the more powerful the upstream. We have demonstrated that $p_1^* = m_1 + \delta_1^{\min}$. It is thus clear that this setting does not favor the upstream if the latter does not have the power to impose a reasonable margin. We represent in Figure 8 the optimal profits Π_1^* , Π_2^* and Π_G^* in function of δ_1^{\min} in the case of $\mu_1 = \mu_2$. We see that Π_1^* is concave in δ_1^{\min} , whereas Π_2^* is decreasing in δ_1^{\min} as expected. If the upstream has the power to increase δ_1^{\min} , then the downstream will quote a shorter DT, which leads to a lower demand. Moreover, the optimal downstream margin decreases. This leads to a lower downstream profit. The optimal upstream profit first increases (since δ_1^{\min} increases) and then the demand becomes too small, which decreases the optimal upstream profit. It is also important to note that the overall SC profit Π_G^* decreases in δ_1^{\min} (Π_G^* is at the maximum when $\delta_1^{\min} = 0$). This is consistent with the findings of the literature. Xiao & Qi (2016) highlight that when the unit wholesale price equals the unit production cost (i.e., $\delta_1^{\min} = 0$ in our case), the decentralized channel is equivalent to the centralized channel.

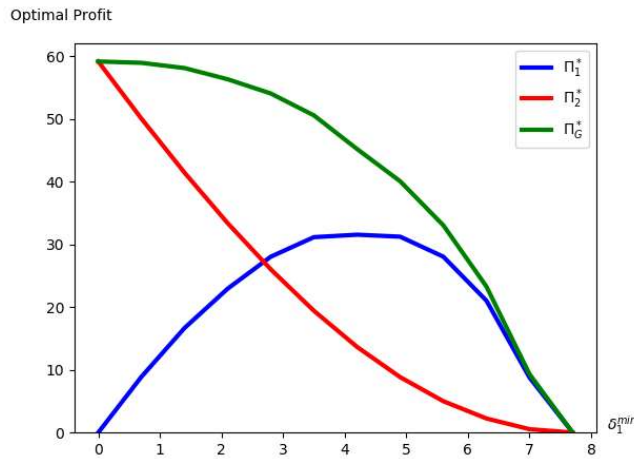


Figure 8. Optimal profits in function of δ_1^{\min}

8 Numerical study

Since closed-form expressions for the expected profits are not obtainable, we perform numerical analysis to gain further insights on SC performance and coordination. We first compare the performances of the different decentralized settings to the centralized case (i.e., when all decisions are undertaken simultaneously by a single decision maker) to investigate the quality of SC coordination in each case. The centralized model is solved numerically with the SLSQP

optimizer. The optimal overall profit obtained under a centralized setting is denoted by Π_C^* . Clearly, the centralized setting gives the highest overall profit that can be generated by the SC. We let Π_G^* denote the overall SC profit obtained at optimality in the decentralized case. In case of setting 1, for instance, Π_G^* is the overall profit obtained with $l = l_1^*$ (see Figures 4 and 5 for illustration). For each setting i , we evaluate the profit gap between Π_G^* and Π_C^* by calculating $Gap(i)_{\Pi_C^*/\Pi_G^*} = \frac{(\Pi_C^* - \Pi_G^*) \times 100}{\Pi_C^*}$. Clearly, the smaller the value of $Gap(i)_{\Pi_C^*/\Pi_G^*}$, the better the SC coordination is. We use the test cases generated according to Table 3.

Table 3. Test cases

Parameter	Values
a	[50, 100] with a step of 10
α	[1, 5] with a step of 1
β	[1, 5] with a step of 1
m_1	[1, 4] with a step of 1
m_2	[1, 4] with a step of 1
μ_1	[10, 60] with a step of 10
μ_2	[10, 60] with a step of 10
δ_1^{\min}	fixed to 2
s	fixed to 95%

We present the results in Table 4 for each of the following test scenarios: $\mu_1 = \mu_2$, $\mu_1 < \mu_2$ and $\mu_1 > \mu_2$. Note that the number of feasible instances varies from one setting to another since specific conditions should be satisfied for each setting to avoid non-realistic solutions as explained in the analytical solving approach. To confirm the interest of our numerical results, we provide the confidence interval which is calculated based on a confidence level of 95%. We can see that the confidence interval is relatively narrow, which means that our estimation of the mean gap is precise.

Table 4. Estimation of $Gap(i)_{\Pi_C^*/\Pi_G^*}$

		Mean (%)	Std. deviation	Confidence interval	Feasible cases
<i>Setting 1</i>	$\mu_1 = \mu_2$	16.87	9.42	(16.71, 17.02)	14360
	$\mu_1 < \mu_2$	12.75	9.66	(12.64, 12.85)	33439
	$\mu_1 > \mu_2$	13.76	9.48	(13.65, 13.86)	33439
<i>Setting 2</i>	$\mu_1 = \mu_2$	2.06	5.65	(1.97, 2.15)	14375
	$\mu_1 < \mu_2$	9.60	25.05	(9.34, 9.86)	35945
	$\mu_1 > \mu_2$	1.63	4.65	(1.58, 1.68)	35930
<i>Setting 3</i>	$\mu_1 = \mu_2$	16.87	9.42	(16.71, 17.02)	14360
	$\mu_1 < \mu_2$	13.76	9.48	(13.65, 13.86)	33439
	$\mu_1 > \mu_2$	12.75	9.66	(12.64, 12.85)	33439

We first recall that settings 1 and 3 lead to the same overall profit when both firms swap their capacities (as demonstrated in Corollary 4). This explains why $Gap(1)_{\Pi_C^*/\Pi_G^*}$ for $\mu_1 < \mu_2$ equals $Gap(3)_{\Pi_C^*/\Pi_G^*}$ for $\mu_1 > \mu_2$. In particular, for $\mu_1 = \mu_2$, we have demonstrated that settings 1 and 3 lead to the same overall profit and are therefore equivalent in terms of SC coordination. It is therefore expected, for $\mu_1 = \mu_2$, that $Gap(1)_{\Pi_C^*/\Pi_G^*} = Gap(3)_{\Pi_C^*/\Pi_G^*}$ as shown in Table 4. The numerical analysis quantifies this profit gap and shows that it equals 16.87% on average (for $\mu_1 = \mu_2$). It is also interesting to note that $Gap(1)_{\Pi_C^*/\Pi_G^*}$ and $Gap(3)_{\Pi_C^*/\Pi_G^*}$ reach the lowest average value when the SC leader has more capacity than the follower (i.e., when $\mu_1 < \mu_2$ for setting 1 and $\mu_1 > \mu_2$ for setting 3). A higher capacity motivates the leader to decide the DTs in a way that favors a higher demand over a higher unit margin, which also benefits to the upstream (as explained earlier) and increases the overall SC profit. While the choice of the SC leader in most related works is justified by market conditions (e.g., a powerful retailer), our results suggest that choosing the SC leader based on operational characteristics (i.e., production capacity allocated by each firm) can be used to achieve a better coordination of the SC. This result is formulated in Observation 1.

Observation 1. *When the SC leader controls the DTs and each firm decides its own price, selecting the firm with the largest capacity to act as a leader can generally improve the SC coordination.*

Furthermore, we see that $Gap(2)_{\Pi_C^*/\Pi_G^*}$ is much smaller than $Gap(1)_{\Pi_C^*/\Pi_G^*}$ and $Gap(3)_{\Pi_C^*/\Pi_G^*}$ in all cases. This means that setting 2 leads to a better coordination of the SC. The reason is that

the double marginalization effect, which is known to reduce the overall profit in decentralized SCs (see e.g. Lee et al. 2000), has here a small impact since the prices are controlled by the downstream, and the upstream firm cannot impose a high margin. We extended the experiments with other values of δ_1^{\min} ranging from 1 to 7 and found that setting 2 still effectively coordinate the SC for realistic values of δ_1^{\min} relative the unit production cost. However, for very high values of δ_1^{\min} (e.g., $\delta_1^{\min} = 6$ or 7), the profit gap relative to the centralized profit becomes very large, which is consistent with what we observed in Figure 8 (Π_G^* is very low for high values of δ_1^{\min}). Thus, for reasonable values of δ_1^{\min} , the case where each firm decides its DT, and prices are controlled by one firm seems to more effectively coordinate the SC than the case where each firm decides its price, and DTs are controlled by one firm. This is an interesting result that cannot be obtained from existing models where only one firm has a DT. This result is formulated in Observation 2. Note also that setting 2 leads to a less effective coordination when $\mu_1 < \mu_2$ ($Gap(2)_{\Pi_C^*/\Pi_G^*} = 9.60\%$ in this case) compared to the cases of $\mu_1 \geq \mu_2$, which contradicts the findings obtained with settings 1 and 3. Therefore, it seems that when the SC leader does not have a full control on DTs, choosing the firm with the largest capacity as a leader does not improve the SC coordination.

Observation 2. *For realistic values of the upstream minimum required margin (relative to the unit production cost), making all pricing decisions by the same firm and allowing each firm to decide its own DT leads to a better SC coordination than making all DT decisions by the same firm and allowing each firm to decide its own price.*

We now investigate how the SC leader can use the DT to achieve a better coordination of the SC (i.e., to increase the overall SC profit). We define Π_G^{\max} as the overall SC profit that can be achieved when the SC leader sets the DT with the objective of maximizing the overall SC profit instead of maximizing its own profit. In case of setting 1, for instance, $\Pi_G^{\max} = \max_{l_1} \Pi_G(l_1)$. We refer the reader to Figures 4 and 5 for illustration (Π_G^* is the overall profit obtained with l_1^* , whereas Π_G^{\max} is the peak of the green curve). For each setting i , we evaluate numerically the profit gap $Gap(i)_{\Pi_G^{\max}/\Pi_G^*} = \frac{(\Pi_G^{\max} - \Pi_G^*) \times 100}{\Pi_G^*}$ and $Gap(i)_{\Pi_C^*/\Pi_G^{\max}} = \frac{(\Pi_C^* - \Pi_G^{\max}) \times 100}{\Pi_G^{\max}}$. Note that $Gap(i)_{\Pi_G^{\max}/\Pi_G^*}$ quantifies the gain that can be achieved if the SC leader manipulates the DTs to coordinate the SC, while $Gap(i)_{\Pi_C^*/\Pi_G^{\max}}$ assesses the quality of the coordination by comparing Π_G^{\max} for each setting i to a same upper bound, which is given by the optimal centralized profit. In these experiments, we consider, for setting 2, the different values of δ_1^{\min} ranging from 1 to 7

with a step of 1. We present the results in Tables 5 and 6 for $Gap(i)_{\Pi_G^{\max}/\Pi_G^*}$ and $Gap(i)_{\Pi_C^*/\Pi_G^{\max}}$, respectively.

Table 5. Estimation of $Gap(i)_{\Pi_G^{\max}/\Pi_G^*}$

	Test scenario	Mean (%)	Std. deviation	Confidence interval	Feasible instances
<i>Setting 1</i>	$\mu_1 = \mu_2$	21.71	13.22	(21.50, 21.90)	14360
	$\mu_1 < \mu_2$	15.13	12.85	(15.00, 15.27)	36000
	$\mu_1 > \mu_2$	15.75	14.30	(15.60, 15.90)	33439
<i>Setting 2</i>	$\mu_1 = \mu_2$	7.28	17.06	(7.18, 7.38)	111878
	$\mu_1 < \mu_2$	5.79	15.85	(5.73, 5.85)	259639
	$\mu_1 > \mu_2$	6.46	18.81	(6.40, 6.52)	281523
<i>Setting 3</i>	$\mu_1 = \mu_2$	21.71	13.22	(21.50, 21.90)	14360
	$\mu_1 < \mu_2$	15.75	14.30	(15.60, 15.90)	33439
	$\mu_1 > \mu_2$	15.13	12.85	(15.00, 15.27)	36000

Table 6. Estimation of $Gap(i)_{\Pi_C^*/\Pi_G^{\max}}$

		Mean (%)	Std. deviation	Confidence interval	Feasible instances
<i>Setting 1</i>	$\mu_1 = \mu_2$	0.05	0.34	(0.05, 0.06)	14360
	$\mu_1 < \mu_2$	0.01	1.00	(0.00, 0.02)	33439
	$\mu_1 > \mu_2$	1.45	3.50	(1.42, 1.49)	33439
<i>Setting 2</i>	$\mu_1 = \mu_2$	0.20	3.42	(0.14, 0.25)	14375
	$\mu_1 < \mu_2$	8.22	25.11	(7.96, 8.48)	35945
	$\mu_1 > \mu_2$	0.07	1.43	(0.05, 0.08)	35930
<i>Setting 3</i>	$\mu_1 = \mu_2$	0.05	0.34	(0.05, 0.06)	14360
	$\mu_1 < \mu_2$	1.45	3.50	(1.42, 1.49)	33439
	$\mu_1 > \mu_2$	0.01	1.00	(0.00, 0.02)	33439

Observing Table 5, we first deduce that a judicious quotation of DT by the SC leader can significantly increase the SC profit relative to Π_G^* . The average gain can reach 21.71% for settings 1 and 3 (with $\mu_1 = \mu_2$) and 7.28% for setting 2 (with $\mu_1 = \mu_2$). We also see that $Gap_{\Pi_G^{\max}/\Pi_G^*}$ reaches its highest value when $\mu_1 = \mu_2$. This can be explained based on Figures 4, 5, 6 and 7. These figures illustrate how the SC leader should quote a longer DT to increase the overall SC profit (as this leads to a greater demand as explained earlier). We see in Table 6 that, under some conditions on capacities, Π_G^{\max} can be very close to Π_C^* in all settings, as we have a mean

gap of 0.01% (with $\mu_1 < \mu_2$), 0.07% (with $\mu_1 > \mu_2$) and 0.01% (with $\mu_1 > \mu_2$) for settings 1, 2 and 3, respectively. Thus, the coordination of the DT quotation between the upstream and the downstream can lead to an overall SC profit that is almost equal to the centralized profit. However, as this implies a lower profit for the SC leader, the leader will not naturally choose this option.

In case of $\mu_1 \neq \mu_2$, however, Table 5 shows that coordination has a relatively smaller effect on improving the decentralized profit Π_G^* with comparison to the cases of $\mu_1 = \mu_2$. The reason is that the SC has less chance to benefit from the additional demand under unbalanced capacities since there is one stage that cannot satisfy the service constraint with this high level of demand. Hence, the disparity between upstream and downstream capacities limits the gain resulting from using DTs to coordinate the SC. Note finally that the profit gap is relatively small in case of setting 2 as this setting is already well coordinated as explained earlier. The above analysis leads to the following result.

Observation 3. *In all settings, the DTs can be effectively used to coordinate the SC. A judicious DT quotation can significantly increase the overall SC profit and even make it very close to the centralized profit. However, the coordination is less efficient in case of unbalanced production capacities (i.e., $\mu_1 \neq \mu_2$).*

The above result highlights the importance of coordinating production capacities between firms to benefit from the DT coordination. This is an important result that cannot be obtained from the related literature since existing works typically consider only one capacitated firm (see Table 1).

Finally, we compare the local profits of each firm under different settings to investigate which setting is more profitable for the firm. Basically, we compare setting 2 to setting 1 to understand which firm benefits from decentralizing the DT decision (as we do in setting 2) instead of decentralizing the pricing (as we do in setting 1). We also compare setting 3 to setting 1 to understand the impact of giving more decision power to the upstream (as we do in setting 3) instead of giving more power to the downstream (as we do in setting 1). Thus, we evaluate numerically the following gaps. We use the instances generated according to Table 3 as well as different values of δ_1^{\min} (when setting 2 is involved). Due to the space constraints, we do not provide here all the experiments but just present some representative results.

- Gap between the optimal profit of the upstream firm under setting 2 and its optimal profit

under setting 1, calculated as $Gap(S_2/S_1)_{\Pi_1^*} = \frac{(\Pi_{1,S_2}^* - \Pi_{1,S_1}^*) \times 100}{\Pi_{1,S_1}^*}$. With $\delta_1^{\min} = 2$, we obtain the average gaps: -22.54% , -26.97% , and -22.26% for $\mu_1 = \mu_2$, $\mu_1 < \mu_2$, and $\mu_1 > \mu_2$, respectively. With $\delta_1^{\min} = 4$, we obtain the average gaps: 23.34% , 20.34% , and 27.80% for $\mu_1 = \mu_2$, $\mu_1 < \mu_2$, and $\mu_1 > \mu_2$, respectively. With $\delta_1^{\min} = 6$, we obtain the average gaps: 49.32% , 49.99% , and 57.59% for $\mu_1 = \mu_2$, $\mu_1 < \mu_2$, and $\mu_1 > \mu_2$, respectively.

- Gap between the optimal profit of the downstream firm under setting 2 and its optimal profit under setting 1, calculated as $Gap(S_2/S_1)_{\Pi_2^*} = \frac{(\Pi_{2,S_2}^* - \Pi_{2,S_1}^*) \times 100}{\Pi_{2,S_1}^*}$. With $\delta_1^{\min} = 2$, we obtain the average gaps: 38.48% , 28.89% , and 31.45% for $\mu_1 = \mu_2$, $\mu_1 < \mu_2$, and $\mu_1 > \mu_2$, respectively. With $\delta_1^{\min} = 4$, we obtain the average gaps: 10.63% , 4.11% , and 5.00% for $\mu_1 = \mu_2$, $\mu_1 < \mu_2$, and $\mu_1 > \mu_2$, respectively. With $\delta_1^{\min} = 6$, we obtain the average gaps: -8.81% , -13.29% , and -13.45% for $\mu_1 = \mu_2$, $\mu_1 < \mu_2$, and $\mu_1 > \mu_2$, respectively.
- Gap between the optimal profit of the upstream firm under setting 3 and its optimal profit under setting 1, calculated as $Gap(S_3/S_1)_{\Pi_1^*} = \frac{(\Pi_{1,S_3}^* - \Pi_{1,S_1}^*) \times 100}{\Pi_{1,S_1}^*}$. We obtain the average gaps: 162.99% , 189.32% , and 211.64% for $\mu_1 = \mu_2$, $\mu_1 < \mu_2$, and $\mu_1 > \mu_2$, respectively.
- Gap between the optimal profit of the downstream firm under setting 3 and its optimal profit under setting 1, calculated as $Gap(S_3/S_1)_{\Pi_2^*} = \frac{(\Pi_{2,S_3}^* - \Pi_{2,S_1}^*) \times 100}{\Pi_{2,S_1}^*}$. We obtain the average gaps: -56.54% , -60.43% , and -59.62% for $\mu_1 = \mu_2$, $\mu_1 < \mu_2$, and $\mu_1 > \mu_2$, respectively.

The above experiments lead to the following final result.

Observation 4. *For realistic values of the upstream minimum required margin (relative to the unit production cost), the upstream firm makes more profit when it decides its price (i.e., setting 1) relative to when it decides its DT (i.e., setting 2), and the downstream firm makes more profit when it controls both prices (i.e., setting 2) relative to when it controls both DTs (i.e., setting 1). However, for relatively high values of the upstream minimum required margin, the upstream firm makes more profit under setting 2, and the downstream firm makes more profit under setting 1. Furthermore, as expected, each firm makes more profit when it leads the SC (i.e., the upstream makes more profit in setting 3 than in setting 1, and the downstream makes more profit in setting 1 than in setting 3).*

When the upstream can guarantee a high minimum required margin δ_1^{\min} , then deciding the DT is better than deciding the price since the control of the local DT allows the upstream

to maximize the demand and thus to guarantee a higher profit since its margin is already high. However, for relatively low values of δ_1^{\min} , the upstream cannot generate a high profit even with a high demand. In this case, the upstream prefers to decide its own price to get a profitable margin and, therefore, setting 1 becomes more profitable. Contrary to the upstream, the downstream firm prefers setting 2 when the upstream has a small power (i.e., δ_1^{\min} is low). The reason is that setting 2 gives the downstream more flexibility to set the prices in a way that maximizes its local profit, and also allows the downstream to benefit from the DT quotation of the upstream who will try to maximize the demand and, therefore, will contribute to improving the downstream profit. When the upstream has more power (i.e., δ_1^{\min} is high), the downstream can get more profit if the upstream decides its own price (i.e., setting 1 instead of setting 2) since the upstream will trade-off its unit margin with the demand and will not choose a high unit margin at optimality. Finally, as expected, each firm makes more profit when it leads the SC compared to the case when the SC is led by the other firm. This result is known in the literature.

9 Conclusion

We studied the DT and pricing decisions in a two-stage decentralized SC facing a price- and delivery time-sensitive demand in a stochastic environment. We defined the problem as a sequential game and considered three settings. First, the downstream, as a Stackelberg leader, decides its price and controls both DTs, and the upstream, as a follower, reacts by deciding its own price. Second, the downstream decides its DT and controls prices, and the upstream reacts by quoting its own DT. Third, the upstream, as a leader, decides its price and controls both DTs, and the downstream, as a follower, reacts by deciding its own price. We characterized analytically the optimal strategy of each firm under each setting. Then, we analyzed the optimal solutions and discussed analytical and numerical insights.

Our study investigated trade-offs that govern the interactions between local DTs, overall DT, prices, demand, and profits. For instance, we found that while the local DTs are always positively correlated at optimality, there are situations where a firm may have interest in decreasing its own price if the other firm increases its price, which differs from the findings of the related literature. Another interesting result is that the follower's expected profit always increases in quoted DT and in mean demand, whereas the SC leader's expected profit is a concave

function in quoted DT and in mean demand. We showed in case of balanced capacity (i.e., same capacity for both firms) that the SC will make the same overall profit and will offer the same product to the customers (in terms of price and DT) regardless of the firm who decides the DTs and acts as a leader. The amount of capacity allocated by each firm impacts the quality of SC coordination. For instance, when the SC leader has a full control on DTs and each firm decides its own price, setting the firm with the largest capacity as SC leader can generally improve the SC coordination. Finally, we showed that the local DTs can effectively coordinate the SC. However, the coordination is less efficient in case of unbalanced production capacities.

Our modeling effort and analysis come with limitations that can provide directions for further research in the area. We assumed that both stages target the same minimum service level. A natural extension would be to study the case where each stage may target a different service level. We also assumed that the optimal strategy for each firm is based on a full information about the capacity of the other firm. It is interesting to consider the case where capacity information is not fully available as this may lead to different trade-offs. Other decision scenarios are also interesting to investigate such as when each stage decides its own price and DT. This scenario, however, is very difficult to solve in a decentralized context.

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Appendix

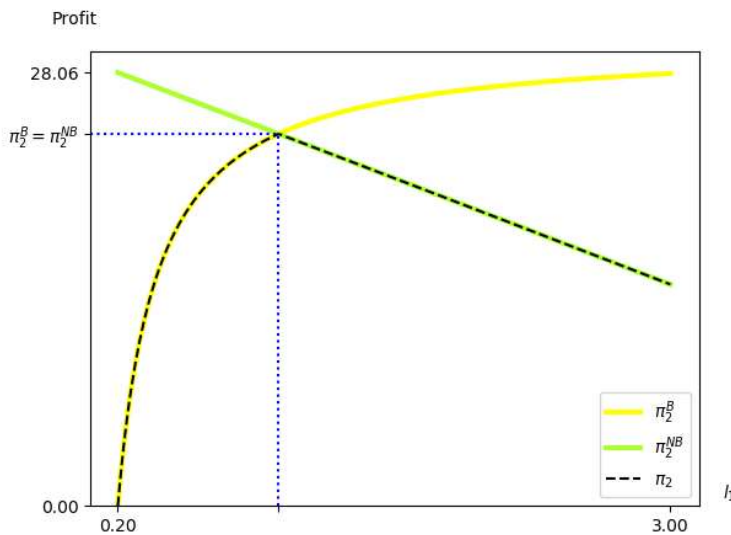


Figure 2. Downstream profit in function of l_1 for given δ_2 (Setting 1)

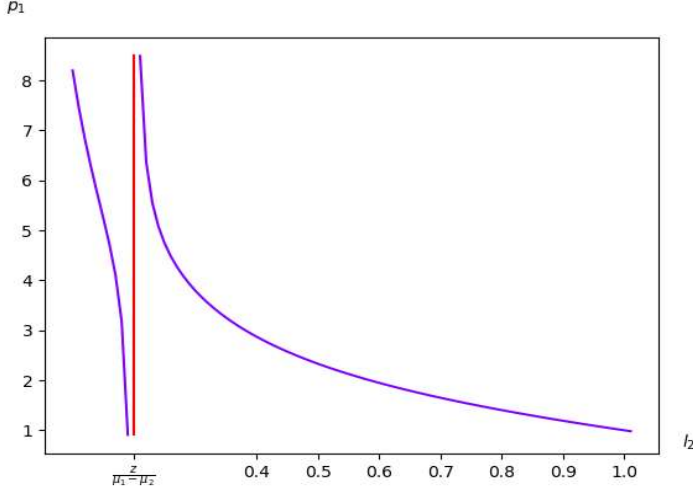


Figure 3. p_1 in function of l_2 with $\mu_2 > \mu_1$ (for given δ_2) (Setting 2)

Solving approach for Setting 3.

We first assume that p_1, l_1 and l_2 are given and solve the downstream model. We obtain $\delta_2^* = \max\left(\frac{a-\alpha(p_1+m_2)-\beta(l_1+l_2)}{2\alpha}, \frac{a-\alpha(p_1+m_2)-\beta(l_1+l_2)-\mu_2+\frac{z}{l_2}}{\alpha}\right)$. We now turn to the upstream model. The service constraint in model (M3.1) is binding and, consequently, it is equivalent to

$$a - \alpha(p_1 + m_2 + \delta_2) - \beta(l_1 + l_2) = \mu_1 - \frac{z}{l_1} \quad (47)$$

If the service constraint in model (M3.2) is binding (i.e., $\frac{a-\alpha(p_1+m_2)-\beta(l_1+l_2)-\mu_2+\frac{z}{l_2}}{\alpha} \geq \frac{a-\alpha(p_1+m_2)-\beta(l_1+l_2)}{2\alpha}$), then $\delta_2 = \frac{a-\alpha(p_1+m_2)-\beta(l_1+l_2)-\mu_2+\frac{z}{l_2}}{\alpha}$ and $\lambda = \mu_2 - \frac{z}{l_2}$ at optimality. Consequently, $l_1 = \frac{zl_2}{(\mu_1-\mu_2)l_2+z}$ at optimality. Otherwise, $\delta_2 = \frac{a-\alpha(p_1+m_2)-\beta(l_1+l_2)}{2\alpha}$ and, consequently, $l_1 = \frac{a-\alpha(p_1+m_2)-\beta l_2-2\mu_1+\sqrt{(a-\alpha(p_1+m_2)-\beta l_2-2\mu_1)^2+8\beta z}}{2\beta}$ at optimality according. Hence, model (M3.1) is equivalent to

$$\text{Maximize}_{p_1 \geq 0, l_2 \geq \frac{z}{\mu_2}} \Pi_1(p_1, l_2) = \begin{cases} \Pi_1^B(p_1, l_2) = (p_1 - m_1) \left(\mu_2 - \frac{z}{l_2} \right) & \text{if } p_1 \text{ and } l_2 \text{ satisfy the binding condition in downstream model,} \\ \Pi_1^{NB}(p_1, l_2) = (p_1 - m_1) \left(\frac{a-\alpha(p_1+m_2)-\beta l_2+2\mu_1-\sqrt{(a-\alpha(p_1+m_2)-\beta l_2-2\mu_1)^2+8\beta z}}{4} \right) & \text{if } p_1 \text{ and } l_2 \text{ satisfy the non-binding condition in downstream model.} \end{cases} \quad (48)$$

We write l_1 in function of p_1 and l_2 and deduce that the binding condition in model (M3.2) is equivalent to $\mu_2 - \frac{z}{l_2} \leq \frac{a-\alpha(p_1+m_2)-\beta l_2+2\mu_1-\sqrt{(a-\alpha(p_1+m_2)-\beta l_2-2\mu_1)^2+8\beta z}}{4}$. This condition is equivalent to $\Pi_1^B(p_1, l_2) \leq \Pi_1^{NB}(p_1, l_2)$. For given p_1 , it can be verified that there is a unique value of l_2 such that $\Pi_1^B = \Pi_1^{NB}$, and we have $\Pi_1^{NB} > \Pi_1^B$ before this intersec-

tion point and $\Pi_1^{NB} < \Pi_1^B$ after it. Moreover, given that Π_1^B is increasing in l_2 and Π_1^{NB} is decreasing, we have $\Pi_1^B = \Pi_1^{NB}$ at optimality. Based on this analysis, we deduce that $\frac{a-\alpha(p_1+m_2)-\beta l_2+2\mu_1-\sqrt{(a-\alpha(p_1+m_2)-\beta l_2-2\mu_1)^2+8\beta z}}{4} = \mu_2 - \frac{z}{l_2}$ at optimality. Therefore, we obtain p_1 in function of l_2 and deduce the upstream model. The rest of the analysis follows immediately.

Proof. (Lemma 1) On the one hand, $\frac{\partial^2 \Pi_1(p_1)}{\partial p_1^2} = -2\alpha < 0$, implying that $\Pi_1(p_1)$ is concave in p_1 , and $\Pi_1(p_1)$ reaches its maximum in $p_1^{NB} = \frac{a-\alpha(\delta_2+m_2-m_1)-\beta(l_1+l_2)}{2\alpha}$ (according to the first derivative condition). On the other hand, service constraint (3) is equivalent to $p_1 \geq \frac{\frac{z}{l_1}-\mu_1+a-\alpha(\delta_2+m_2)-\beta(l_1+l_2)}{\alpha}$. We let $p_1^B = \frac{\frac{z}{l_1}-\mu_1+a-\alpha(\delta_2+m_2)-\beta(l_1+l_2)}{\alpha}$. Thus, given the concavity of $\Pi_1(p_1)$, the optimal price $p_1^* = \max(p_1^B, p_1^{NB})$, where p_1^B and p_1^{NB} are, respectively, the optimal prices in binding and non-binding situations.

Furthermore, the service constraint is binding if and only if $p_1^B \geq p_1^{NB}$, which is equivalent to $\beta l_1^2 - (a - \alpha(\delta_2 + m) - 2\mu_1 - \beta l_2) l_1 - 2z \leq 0$.

It is highlighted that the obtained prices are positive and yield positive demand. Thus, $p_1^{NB} \geq 0$ since the DTs and margin quoted by the downstream will verify $l_1 + l_2 \leq \frac{a-\alpha(\delta_2+m)}{\beta}$ (otherwise, it is not possible to get a positive demand for any value of $p_1 \geq m_1$). If the binding situation is preferred, then $p_1^B \geq 0$ since we necessarily have $p_1^B \geq p_1^{NB}$ in this case. With respect to demand, first recall that demand is positive if and only if $p_1 \leq \frac{a-\alpha(\delta_2+m_2)-\beta(l_1+l_2)}{\alpha}$. Given that $l_1 \geq \frac{z}{\mu_1}$ (otherwise, the upstream model cannot be feasible), then $p_1^B \leq \frac{a-\alpha(\delta_2+m_2)-\beta(l_1+l_2)}{\alpha}$. Moreover, it can be verified that $p_1^{NB} \leq \frac{a-\alpha(\delta_2+m_2)-\beta(l_1+l_2)}{\alpha}$ since $l_1 + l_2 \leq \frac{a-\alpha(\delta_2+m)}{\beta}$. ■

Proof. (Lemma 2) Suppose that we have at optimality δ_2^* , l_1^* and l_2^* , such that $(\mu_2 - \lambda^*) l_2^* > z$. Substituting p_1^* by its expression, this inequation becomes equivalent to $\left(\frac{z}{l_1^*} + \mu_2 - \mu_1\right) l_2^* > z$ in a binding situation, and $\left(\mu_2 - \frac{a-\alpha(\delta_2^*+m)-\beta(l_1^*+l_2^*)}{2}\right) l_2^* > z$ in a non-binding situation.

In a binding situation, we can decrease l_2^* to l_2' while keeping δ_2^* and l_1^* constant until we obtain $\left(\frac{z}{l_1^*} + \mu_2 - \mu_1\right) l_2' = z$. The new solution $(\delta_2^*, l_1^*, l_2')$ is still feasible, and $\Pi_2(\delta_2^*, l_1^*, l_2') > \Pi_2(\delta_2^*, l_1^*, l_2^*)$ (since demand has increased), which is impossible.

In a non-binding situation, we can decrease l_1^* to l_1' while keeping δ_2^* and l_2^* constant until we obtain $\left(\mu_2 - \frac{a-\alpha(\delta_2^*+m)-\beta(l_1'+l_2^*)}{2}\right) l_2^* = z$. The new solution $(\delta_2^*, l_1', l_2^*)$ is still feasible, and $\Pi_2(\delta_2^*, l_1', l_2^*) > \Pi_2(\delta_2^*, l_1^*, l_2^*)$, which is impossible. Hence, in both cases, the service constraint in the downstream model is binding. ■

Proof. (Lemma 3) Case of binding situation in the upstream model. In this case, $p_1 = \frac{\frac{z}{l_1}-\mu_1+a-\alpha(\delta_2+m_2)-\beta(l_1+l_2)}{\alpha}$. As constraint (7) is binding, we have $(\mu_2 - a + \alpha(p_1 + m_2 + \delta_2) + \beta(l_1 + l_2)) l_2 = z$ at optimality. Replacing p_1 with its expression in the previous equation, it comes that

$l_2 = \frac{z}{l_1 - \mu_1 + \mu_2}$ at optimality. The expressions of p_1 and λ as a function of δ_2 and l_1 can then be obtained by standard calculus.

Case of non-binding situation in the upstream model. In this case, $p_1 = \frac{a - \alpha(\delta_2 + m_2 - m_1) - \beta(l_1 + l_2)}{2\alpha}$.

Given that $(\mu_2 - a + \alpha(p_1 + m_2 + \delta_2) + \beta(l_1 + l_2))l_2 = z$ at optimality, we substitute p_1 by its value, and the previous equation becomes equivalent to $\beta l_2^2 - (a - \alpha(\delta_2 + m) - \beta l_1 - 2\mu_2)l_2 - 2z = 0$. The discriminant of this equation is positive. Hence, the equation has two roots. Since one of them is negative, the value of l_2 is given by the positive root,

namely $\frac{a - \alpha(\delta_2 + m) - \beta l_1 - 2\mu_2 + \sqrt{(a - \alpha(\delta_2 + m) - \beta l_1 - 2\mu_2)^2 + 8\beta z}}{2\beta}$. We then obtain the expressions of p_1

and λ as a function of δ_2 and l_1 . ■

Proof. (Proposition 1) l_1^* is obtained from (14). We use Lemma 3 to determine l_2^* and p_1^* (we can use either the binding or the non-binding properties and obtain the same results since $\Pi_2(l_1)$ is calculated in the intersection between these two situations). We then deduce $\delta_1^* = p_1^* - m_1$. Furthermore, δ_2^* is deduced from equation (11) and $p_2^* = p_1^* + m_2 + \delta_2^*$. ■

Proof. (Lemma 4). First recall that $l_1^*(\delta_2, l_2, p_1) = \frac{a - \alpha(p_1 + m_2 + \delta_2) - \beta l_2 - \mu_1 + \sqrt{(a - \alpha(p_1 + m_2 + \delta_2) - \beta l_2 - \mu_1)^2 + 4\beta z}}{2\beta}$.

Given that $\lambda = \mu_1 - \frac{z}{l_1}$ at optimality, then demand increases in $l_1^*(\delta_2, l_2, p_1)$. For fixed δ_2 and p_1 , it can be easily verified that l_1^* is decreasing in l_2 , which implies that demand is also decreasing in l_2 (for fixed δ_2 and p_1).

We can now demonstrate the result of Lemma 4 by contradiction. Assume that we have an optimal solution $(\delta_2^*, l_2^*, p_1^*)$ such that $1 - e^{-(\mu_2 - \lambda^*)l_2^*} > s$. We can decrease the DT from l_2^* to l_2' while keeping δ_2^* and p_1^* constant until we obtain $1 - e^{-(\mu_2 - \lambda')l_2'} = s$. Solution $(\delta_2^*, p_1^*, l_2')$ is feasible and leads to a higher demand, which implies that $\Pi_2(\delta_2^*, p_1^*, l_2') > \Pi_2(\delta_2^*, p_1^*, l_2^*)$. This is impossible. Consequently, the service constraint is binding. ■

Proof. (Remark 1) If $\mu_2 \leq \mu_1$, then the discontinuity occurs for negative values of l_2 . If $\mu_2 > \mu_1$, then we must consider only the values of l_2 such that $l_2 < \frac{z}{\mu_2 - \mu_1}$. On the one hand, we have at optimality $\lambda = \mu_2 - \frac{z}{l_2}$ and, on the other hand, $\lambda < \mu_1$ according to the steady-state condition, which implies that $l_2 < \frac{z}{\mu_2 - \mu_1}$ at optimality. ■

Proof. (Remark 2) If $p_1(\delta_2, l_2^{\max}) > m_1 + \delta_1^{\min}$, then it is easy to ascertain that constraint (30) cannot be satisfied. Consequently, $(\delta_2, l_2^{\max}, p_1(\delta_2, l_2^{\max}))$ is not a feasible solution. In this case, the longest feasible DT (if it exists) is obtained when constraint (30) is binding. This implies that the optimal demand is equal to zero, and the problem is not relevant. ■

Proof. (Proposition 2) Follows from the analysis of setting 2. ■

Proof. (Proposition 3) Methodology similar to setting 1. ■

Proof. (Corollary 1) In all settings, $l_1 = \frac{zl_2}{(\mu_1 - \mu_2)l_2 + z}$ at optimality, and we have $l = l_1 + l_2$. DTs are therefore positively correlated. For prices, it can be verified, for instance in case of setting 1, that p_1^* increases in DT and p_2^* decreases in DT. ■

Proof. (Corollary 2). In case of setting 1, for instance, we have $\lambda = \mu_1 - \frac{z}{l_1}$ at optimality, which implies that demand increases in DT. Same for the other settings. ■

Proof. (Corollary 3) In case of setting 1, for instance, we deduced that $\Pi_1(l_1) = \frac{1}{\alpha} \left(\mu_1 - \frac{z}{l_1} \right)^2$. Hence, the upstream profit increases in l_1 and therefore increases in l_2, l and λ (as they are positively correlated). Given that $\lambda = \mu_1 - \frac{z}{l_1}$ at optimality, $\Pi_2(l_1)$ can be expressed in function of λ as $\Pi_2(\lambda) = \lambda \left[\frac{1}{\alpha} \left(a - \frac{\beta z}{(\mu_1 - \lambda)} - \frac{\beta z}{(\mu_2 - \lambda)} - 2\lambda \right) - m \right]$. It can be easily verified that $\Pi_2(\lambda)$ is concave (and therefore $\Pi_2(l_1)$ is concave). Same approach for the other settings. ■

Proof. (Corollary 4) Consider setting 1 with capacities μ_1 and μ_2 for upstream and downstream, respectively. According to Proposition 1, the optimal upstream DT is given by maximizing $\left(\mu_1 - \frac{z}{l_1} \right) \left[\frac{1}{\alpha} \left(a - \beta l_1 - \frac{\beta z}{(\mu_2 - \mu_1 + \frac{z}{l_1})} - 2 \left(\mu_1 - \frac{z}{l_1} \right) \right) - m \right]$ and $l_2^* = \frac{zl_1^*}{z + (\mu_2 - \mu_1)l_1^*}$. We now consider setting 3 with capacities μ_2 and μ_1 for upstream and downstream, respectively. According to Proposition 3, the optimal downstream DT is given by maximizing

$\left(\mu_1 - \frac{z}{l_2} \right) \left[\frac{1}{\alpha} \left(a - \beta l_2 - \frac{\beta z}{(\mu_2 - \mu_1 + \frac{z}{l_2})} - 2 \left(\mu_2 - \frac{z}{l_2} \right) \right) - m \right]$ and $l_1^* = \frac{zl_2^*}{z + (\mu_2 - \mu_1)l_2^*}$. Thus, the optimal upstream DT (respectively, downstream DT) for setting 1 is equal to the optimal downstream DT (respectively, upstream DT) for setting 3. Consequently, the overall DT l^* is the same in both cases.

Under setting 1, the final price is $\frac{1}{\alpha} \left[a - \beta l_1^* - \frac{\beta z}{(\mu_2 - \mu_1 + \frac{z}{l_1^*})} - \left(\mu_1 - \frac{z}{l_1^*} \right) \right]$. Under setting 3, it equals $\frac{1}{\alpha} \left[a - \beta l_2^* - \frac{\beta z}{(\mu_1 - \mu_2 + \frac{z}{l_2^*})} - \left(\mu_2 - \frac{z}{l_2^*} \right) \right]$. Given that μ_1, μ_2 and l_1^* of setting 1 are respectively equal to μ_2, μ_1 and l_2^* for setting 3 when we swap capacities, we deduce that the final price is the same in both cases.

Similarly, considering the objective function of each stage under each setting and the fact that μ_1, μ_2 and l_1^* of setting 1 are respectively equal to μ_2, μ_1 and l_2^* for setting 3 when we swap capacities, we deduce that the upstream and downstream profits are also swapped. ■