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# **A maximum-flow network interdiction problem in an uncertain environment under information asymmetry condition: Application to smuggling goods**

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## **Abstract**

We study the interdiction of smuggling network that arranging the activities of the police in order to successfully interdict criminals in smuggling goods. This work contributes to the literature of maximum flow network interdiction problems by addressing asymmetric information, uncertain conditions, multi commodity, and with multiple sources (origins) and sinks (destinations). Information Asymmetry realistically occurs due to incomplete information of interdictor (police) and operator (smuggler) about each other's performance, which is adapted from the real-world condition. We propose two mixed-integer programming models by reformulating a Min–Max bi-level mathematical model. In the first model, the type of interdiction is discrete (zero and one), while in the second model, the interdiction is assumed continuous, meaning that the partial interdiction is possible. The asymmetry type of the smuggler's information towards the police have formulated through a linear function while the asymmetry of the police information to the smuggler is formulated using an uncertain parameter through a two-stage stochastic programming framework. To solve the first model, an innovative exact hybrid method is proposed combining of a Decomposition Method and Progressive Hedging Algorithm (DM-PHA). An augmented Karush-Kuhn-Tucker (KKT) method is also used to solve the second model. Several sensitivity analyses are then conducted, and the results demonstrate the applicability and effectiveness of the proposed models as well as the solving approach. It is also shown that the proposed models can be used as a suitable approach in uncertain environment and under asymmetric information to determine the optimal interdiction decisions of police to prevent further smuggling.

**Keywords:** The maximum flow network interdiction; Two-stage stochastic programming; decomposition-PHA algorithm; Smuggling goods; Karush-Kuhn-Tucker conditions (KKT).

## **1. Introduction**

Today, smuggling is considered as a social and economic issue. Obviously, the extension of this plight can annihilate the social and economic foundations of society. Illicit trade activities such as the delivery of illegal goods and services have numerous negative socio-economic effects (Basu G., 2014). These activities can threaten public health (e.g., fake medicine and drugs), the environment (e.g., endangered wildlife), human rights (e.g., slavery and prostitution), security (e.g., financing terror operations), cultural heritage (e.g., trafficking of cultural objects), economy (e.g., sale of counterfeit products), and even government income (e.g., tax revenue and legitimate employment) (Jabarzare et al., 2020). Although it is difficult to accurately estimate the scope and scale of illicit trade due to its secretive

nature, the World Economic Forum (World Economic Forum, 2015) estimated that this grey economy is worth US \$650 billion dollars and is expected to grow.

To confront the issue of smuggling, law enforcement officials are constantly monitoring and interdicting these illegal operations. However, law enforcement officials are more concerned with either long-term goals (e.g., minimizing the success of trafficking operations) as a result of interdiction policies, or deciding how to most effectively apply their limited budget and resources. The high importance of this issue has led us to study the distribution of smuggling goods and provide a targeted approach to control smuggling using network interdiction Problem.

In an interdiction problem, an agent attempts to limit the actions of an adversary operating on a system (e.g., a network) by intentionally disrupting certain components of the system. Such problems are usually modeled in the framework of Stackelberg game and can be formulated as bi-level optimization problems. Based on the literature for network interdiction problems, law enforcement officials are the interdictor (leader) and smugglers are the operator (follower). Operator seeks to optimize a specific goal in the network. He wants to use the structure of the network in his own interest and the best way to achieve goals such as sending the maximum flow, moving on the shortest path, transferring several products at the lowest cost or moving with the least possible arrest. The other player, called the leader or the interdictor, wants to influence the network in a way that prevents its optimal use of the network before any action and decision by the operator. To do this, he can increase the cost of a unit of flow on arcs or reduce the capacity of an arc. He is also able to destroy certain components of the network, such as arcs or nodes, and, by complete or even partial removing them from the network, lead to a change in the structure of the network. However, the interdictor to achieve his goal has limited resources for change and destruction (Smith et al., 2013).

Information asymmetry occurs frequently in network problems with more than one operator, since each operator has his/her own perception of the network and makes decisions, accordingly. Despite the fact that the asymmetric information is a natural and fundamental assumption in any real world semi-structured problems, this assumption is neglected in the literature due to the complexity added to the classic form of the problems in both modeling and solving perspectives. In this study, an extension of the maximum flow network interdiction problem is studied, where the information asymmetry between the interdictor and the operator is considered. The information asymmetry is formulated in two different classes of interdiction models; discrete and continuous. In the former model, the type of interdiction is zero and one, while in the latter; the interdiction is supposed to be fractional, ranging from non-interdiction to complete interdiction.

Hence, a smuggling network is considered in which the smuggler tries to transfer the smuggling goods from the origins (near the boundary lines) to the destinations (central cities) through a connected set of roads. The smuggler has a limited information about the number and location of the police checkpoints

(interdiction) and this is called hereafter the asymmetric information of smuggler toward police, and formulated by using a linear function. It is assumed that the smuggler tries to compensate for his lack of information by bribery or other illegal actions such as unauthorized access to police databases, hiring local informant to buy information, hacking system, etc. On the other hand, the police utilize their infrastructures and other facilities to disrupt the flow of the smuggling goods through the networks as much as possible. Due to the resource limitations, it is very important for the police that on which parts of the network, they establish the checkpoints. It is quite difficult for them to distinguish which routes (arcs) the smugglers are deciding to cross the smuggling goods. The lack of information of the police about the smuggler's hangouts is called hereafter the asymmetric information of police towards smuggler and formulated as an uncertain parameter through a two-stage stochastic programming framework. Therefore, in this network, law enforcement and smugglers do not have the same information about the network. The police have the lack of information about the smugglers hangouts and the smuggler has the lack of information about the police checkpoints, meaning that, the police and smugglers are in an asymmetric conflict.

In order to compensate for the lack of information, each player has its own strategy; the police utilize a scenario based approach and manage their limited facilities accordingly to mitigate the risk of smuggler's deception operations. On the other side, the smuggler tries to mitigate the risk of arrest by the police, and fill his lack of information about the police checkpoints through accessing the police confidential information sources (e.g. bribery, hacking, etc., each obviously has its own price) to avoid high-risk roads (in his opinion). This way, the smuggler first estimates the police checkpoints (numbers and locations) and then make effort to commit smuggling of goods, accordingly. He also attempts to pay for a better estimation (bribery, hacking, ...). On the other hand, the police are unaware of the smuggler's preliminary estimates, and they attempt to interdict the network by placing the appropriate number of checkpoints in high-risk roads (in his opinion) based on a set of discrete scenarios. Each scenario describes the probability with which each road can be the haunt of smuggler. These scenarios are wisely defined by the police subject matter expert.

To the best of our knowledge this study is the first effort to extract the exact concept of information asymmetry in combating smuggling of goods and integrate it in discrete and continues interdiction models which are fall within the two-stage bi-level mathematical models. The complicated nature of the two-stage bi-level models pushed us to develop two appropriate solution methods which enable us to provide high quality solutions in reasonable time.

The remainder of this paper is organized as follows: the next section reviews the related works on the interdiction network problems to derive the research gap and emphasizes the main contributions of this paper. Section 3 provides the problem description and mathematical models for the discrete and continuous form of interdiction. Section 4 details the solution algorithms for the proposed models. In

Section 5 the proposed models are implemented in two test problems and the applicability and efficiency of our models as well as the solving approaches are examined and several sensitivity analyses are conducted. Finally, the conclusions of this research are presented in section 6 alongside with some promising directions for further research in this area.

## **2. Literature review**

The network interdiction is a prominent method to distinguish the vital components of a network in critical and security applications to increase the survivability of these networks. The first research on interdiction network problems began with regard to military applications, including how to prevent enemy forces from going on the network of routes. Due to its high efficiency in many real-world problems, interdiction network problems have attracted the attention of researchers and have been developed to apply in various fields (Dai and Poh, 2002) including but not limited to hardening power grids encountering disruptions by natural disasters (e.g., hurricanes) (Yuan et al., 2016), interdicting nuclear smugglers (Sullivan et al., 2014), combating illegal drug trafficking by considering both information and physical flows (Baycik et al., 2018), rail-truck intermodal transportation (Sarhadi et al., 2017) and control infectious diseases (Collado and Papp, 2012).

According to the type of the operator's activity, the network interdiction problem is classified to four general categories in the literature; 1) the maximum flow network interdiction problem (Wollmer, 1964; McMasters and Mustin, 1970; Wood, 1993; Ghare et al., 1971), 2) shortest path network interdiction problem (Lei et al., 2018; Song and Shen, 2016; Taha, 1975), 3) minimum cost flow network interdiction problem (Seifi et al., 2017; Zhang et al., 2017), and 4) facility assignment interdiction problem (Aksan and Aras, 2012).

### *2.1. Maximum flow network interdiction problem*

In this article, the Maximum Flow Network Interdiction Problem (MFNIP) is considered that minimizes the maximum flow of the origins to destinations through a capacitated network by interdicting network arcs with limited interdiction resources. One of the earliest works in this area is carried out by Wollmer (1964) whose objective was to remove  $k$  arcs that cause the greatest decrease in the maximum flow from the origin to the destination. McMasters and Mustin (1970) extended this study to allow partial decreases in the capacities of the arcs. Wood (1993) investigated the continuous maximum flow network interdiction problem in a directed and capacitated graph with a limited interdiction budget. Wood shows that even if the cost of interdiction of all arcs is equal, the complexity degree is NP-complete. He formulated this problem as a simple integer programming model with a set of Valid Inequalities (VIs) and showed that the value of the objective function corresponding relaxed linear problem has a quantitative

difference with the objective function of the integer programming problem. In Altner et al. (2010), a new set of VIs for the problem was presented and shown that the value of the optimality gap obtained from the correct integer formulation, even with new VIs, remains large.

The multi-commodity maximum flow network interdiction problem in discrete and continuous interdiction models has also been investigated in the literature. In Lim and Smith (2008), an integer-programming model was presented for discrete interdiction by introducing a finite coefficient in the objective function. To solve the problem for continuous setting, an optimal division algorithm with an innovative algorithm was developed.

The bi-objective interdiction problem was also studied in Royset and Wood (2007). The purpose of this study was to determine the effective boundary in the interdiction problem of maximum flow with two objective functions aiming at minimization of both maximum flow and cost of interdiction. For this purpose, a new algorithm was developed that provides effective solutions for solving a sequence of single-objective problems through the combination of Lagrange relaxation and branch and bound methods. The maximum flow network interdiction considering time factor is dynamically examined by the researchers in the literature. Rad and Kakhki (2013) introduced the dynamics of interdiction problem, taking into account the parameter of permitted flow time for each arc. Enayaty-Ahangar (2019) developed a large-scale optimization approach for solving an application of a multi-period bi-level network interdiction problem (NIP). In this class of multi-period NIP, interdiction activities must be scheduled to minimize the cumulative maximum flow over a finite time horizon.

In the maximum flow network interdiction problem under uncertainty conditions, the successful interdiction of arcs is random and follows a pre-defined distribution function, while the other network parameters are deterministic. In this case, the interdiction objective is to minimize the expected flow rate. In the research of Garces et al. (2009), the interdiction problem aimed to minimize the highest random flow under heterogeneous risk. Five two- and three-level models for different risk areas were proposed. Each model was formulated in a mixed integer programming and used in real networks to show the interactions between the interdictor and the operator under different levels of risk-taking. Fischetti et al. (2018) investigated this problem under uncertainty and proposed a solution that dynamically divides the solution space into smaller regions.

## *2.2. Information asymmetry in network interdiction*

The information asymmetry is usually seen in network problems that have more than one operator, while each operator has its own perception of the network and decision-making on it (Bayrak & Bailey (2008)). A well-known application of information asymmetry is the problem of positioning sensors. In sensor location problems, the operator tries to track location of the sensors so that it can easily navigate from the origin to the intended destination, while the interdictor wants to increase the tracking and arrest the

operator. There is a traceability probability, which increases with sensor placement. In the meantime, the operator chooses a route that is less likely to be tracked (Golden, 1978). Generally, the operator does not have the exact information about the interdiction capacity of the sensors and can estimate it based on the system's initial information, while the interdictor is fully aware of this capacity. Asymmetric information is rarely discussed in the literature of the network interdiction problem (Bayrak & Bailey, 2008, Sullivan, et al., 2014). Bayrak & Bailey (2008) proposed a shortest path problem where the asymmetric information is formulated by uncertainty. Sullivan et al. (2014) proposed a model for securing a border. Their model minimizes the evasion probability of the smuggler. The fact that the smugglers' ordering of the checkpoints differs from the true ordering is assumed as the information asymmetry. Recently, Nguyen and Smith (2021) proposed a shortest path model in which the cost parameter is supposed to be uncertain and under the asymmetric condition. However, in real world applications, the information of the players about each other is always imperfect and is not limited to cost parameter. Hence the need for considering this challenging concept is quite necessary especially in cases like smuggling, terrorist attacks, etc. In this paper, an extended maximum flow network interdiction model is developed and a new formulation for asymmetric information is proposed. The information asymmetry of the smuggler toward the police checkpoints is formulated by the help of a linear function with which the smuggler tries to improve its primary estimation of the number and locations of the police checkpoints, by illegal actions. On the other hand, the asymmetric information of the police towards smuggler is formulated as a two-stage stochastic framework with which the law enforcement officials' subject-matter experts wisely define some discrete scenarios under which they estimate the response of the smuggler.

### *2.3. Network interdiction problems with uncertainty*

The interdiction models in the network with uncertainty were initially presented by Cormican et al. (1998) in which the success of binary interdiction attacks and arc capacities were uncertain. In the work of Morton (2007), a nondeterministic interdiction was presented to prevent the nuclear smuggling in which the cost of arcs indicates the probability of tracing, and the interdiction objective is to increase this probability in all possible paths. This model was converted into a deterministic model using a logarithmic transformation.

Pan and Morton (2010) studied a stochastic problem where the source nodes and sink nodes are random. The authors formulated this problem as a two-stage stochastic program and proposed three different solution methods. Song and Shen (2016) studied a stochastic interdiction problem where the arc length is uncertain. The authors used Conditional Value at Risk (CVaR) and worst-case CVaR risk measures to investigate routing policies that minimize the expected travel time. Zhang et al. (2018) considered another

stochastic NIP in which the capability of interdiction is supposed to be uncertain. This problem was formulated as a two-stage stochastic program and applied in a case study of the Arizona–Mexico border.

In this paper, we formulated the asymmetric information of the police towards smuggler as an uncertain parameter and proposed a scenario-based two-stage stochastic programming framework.

#### *2.4. Smuggling interdiction*

The total volume of global trade is staggeringly large. Increases in trade volume have been accompanied both by international treaties and by increases in supply chain efficiency. This trade is now the life blood of the global economy. However, these increases in global trade have also provided the means for smugglers to traffic illicit materials in to a target country. International illicit trade activity involves the cross-border economic exchange of prohibited goods and services, ranging from drug trafficking (O'Reilly et al., 2020), smuggling of migrants, human trafficking (Munro-Kramer, 2020), intellectual property infringement, money laundering (Soudijn, 2016), trafficking in firearms (Hughes, 2020), endangered wildlife (Soon J, 2018) and stolen cultural artifacts (World Customs Organization 2009a and 2009b).

Policymakers need models and decision support tools that are both reliable and germane to inform the choices that they make, but such assets are few and far between (Bakker et al., 2020). Game theory has been widely used to study illicit trade activities mathematically. Various analyses used deterministic strategic games (Sandler T, 2003), stochastic games (Kardes E, 2005), evolutionary games (Artigues et al., 2003), signaling games and dynamic games (Overgaard PB, 1994). Many of these games assumed common knowledge or symmetric information between players (Sandler T, 2006; Powell R. 2007). Bakker and et al, (2020), have developed a multi-game model to account for both security-related and economic aspects of counter-smuggling interdiction efforts. They used different games to represent different types of interactions, and these games are then coupled to each other to form an integrated model.

Despite the similarities in the literature, different underlying assumptions exist. Using key features of network interdiction problems, Table 1 classifies the most relevant studies and positions our work amongst them. The major contributions of our study are summarized below:

1. The problem under consideration captures the network interdiction with information asymmetry, multiple commodities in a network and multiple sources and sinks. To the best of our knowledge, the combination of these assumptions, along with the uncertainty condition has not yet been studied in the literature. The asymmetric information of the smuggler toward police and information asymmetry of the police toward smuggler are formulated by a linear function and two-stage stochastic framework, respectively.

2. We present two novel two-stage bi-level mixed-integer programming model with a mini-max objective function. In the first model, the interdiction is applied discretely on arcs and the interdiction variables are set to zero and one while in the second model the interdiction is partial and the arc will not be deleted if the interdiction is not complete.
3. Since this problem belongs to the NP-complete class and is dropped into the challenging category of two-stage bi-level mathematical models, two innovative exact methods are presented; i) a hybrid method based on decomposition and progressive hedging algorithms and ii) an augmented KKT based reformulation with fewer decision variables and constraints.

In **Table 1**, we present an overview of the main interdiction network problem investigations related to our framework. The Table shows that several studies were concerned with the interdiction problems. Despite the fact that the realistic assumption of “interdiction” has recently attracted the attention of several researchers (e.g.,\_Lei et al., 2018; Zhang et al., 2017; Seifi et al., 2017; Nandi and Medal, 2016), the research literature still misses studies investigating the network interdiction with information asymmetry, and only scarce studies (Song et al., 2016; Nandi and Medal, 2016) have applied stochastic network interdiction. This paper aims to bridge over the aforementioned gaps by developing an interdiction model to formulate the two-stage stochastic maximum-flow network interdiction problem with information asymmetry in an uncertain environment to reflect the challenging issues of reality in to the mathematical programming models.

Author(s)	Stochastic	Interdiction network	Interdiction type	Objective function of	Asymmetric	Solution methodology	Application
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**Table 1.** Recent studies on the network interdiction literature

		Maximum flow	Shortest path	Minimum cost	Facility assignment	Discrete	Continuous				
Nguyen and Smith (2021)	✓		✓			✓		Maximize the shortest path	✓	Heuristic algorithm	Smart grid protection, cyber-physical system security
Sreekumaran et al. (2020)			✓			✓	✓	Maximize the shortest path		Lemke's algorithm/ Heuristic algorithm	Military and security
Kosanoglu et al. (2020)	✓				✓	✓		Maximize the minimum cost		Reformulation	Transportation security
Xiang et al. (2020)			✓			✓		Maximize the shortest path		BAB method, Matlab2014a	Terrorist attacks
Shan and Zhuang (2020)	✓			✓		✓		Maximize the minimum cost		Heuristic algorithm	Cyber threats
Lei et al. (2018)			✓			✓		Maximize the shortest path		Reformulation / Python	Smuggling & military logistics
Seifi et al. (2017)				✓		✓		Maximize the minimum cost		Duality, decomposition and reformulation /GAMS	Terrorist attacks, earthquakes, nuclear smuggling, military planning, water resources management, loss control and supply chain
Zhang et al. (2017)				✓		✓		Maximize the minimum cost		Benders decomposition / MATLAB	The border between the two countries (United States and Mexico)
Chestnut et al. (2016)		✓				✓		To minimize the maximum flow		Approximation algorithm	Preventing the distribution of drugs & the spread of infection in a hospital
Song et al. (2016)	✓		✓			✓		Maximize the shortest path		Branch and cut algorithm / CPLEX 12.5	Nuclear trafficking and analysis of vital infrastructure
Nandi and Medal (2016)	✓	✓				✓		Minimize the maximum flow		Heuristic algorithm	Preventing the spread of infection and the virus
Aksen et al. (2014)					✓		✓	Maximize the minimum cost		Heuristic / CPLEX 12.1	Terrorist, rail networks, and other basic infrastructure
Sullivan et al. (2014)	✓					✓		minimizes the smuggler's evasion probability	✓	CPLEX 12.2	Securing a border
Malaviya et al. (2012)		✓				✓		Minimize the maximum flow		CPLEX 12.0	Prevent distribution of drugs at the city level
Bayrak and Bailey (2008)	✓		✓			✓		Maximize the shortest path	✓	CPLEX 9.0	Sensor placement problem
This Research	✓	✓				✓	✓	Minimize the maximum flow	✓	CPLEX 12.8	Preventing smuggling of goods

### 3. Model formulation

This section first describes the investigated interdiction network problem, then introduces the notations to be used in this study, and finally presents two mathematical models under discrete and continuous settings.

#### 3.1. Problem definition

The problem under investigation is a maximum flow network interdiction problem based on a relationship network with the following major characteristics: multiple sources and sinks, uncertainty conditions, information asymmetry (between interdictor and operator), and multiple commodities. The smuggler tends to maximize the flow of several types of trafficking commodities, while the law enforcement officials' goal is to minimize the smuggler's success. The interdictor (law enforcement officials) have a limited budget available and the interdiction is accompanied by actions such as the constructions of inspection stops, subtle patrols and special operations. Smugglers also have limited budget to protect the network. Their budget is used to create the passages and increase the rate of smuggling goods capacity by using underground tunnels, and/or drones, and improve their initial estimate of the number and location of police checkpoints through hiring local informants, bribery, hacking, unauthorized access to the police confidential databases, etc. These assumptions are captured from reality and incorporated into the modeling of the problem.

In this network, the law enforcement and smugglers do not have the same information about the network. Smugglers cannot correctly estimate the number and locations of interdiction in each path, so they first estimate it and then try to improve their primary estimation by spending (e.g. by bribery, unauthorized access to the police confidential databases, etc.). On the other hand, law enforcement officials are also unaware of the routes used by the smuggler. Although the police are generally aware of that the smuggler always tries to buy the police information, they are not sure which checkpoints' information is actually leaked. Hence, due to the budget limitation (and consequently the limit number of checkpoints), they try to increase the arrest probability (interdiction of the smuggling flow) by defining a set of discrete scenarios with each of which an occurrence probability is associated. Each scenario describes the possible reactions of the smugglers, for instance the arcs they probably path the smuggling goods, and/or the checkpoints which their information are probably leaked. The police therefore arrange their actions based on the given scenarios. These scenarios are wisely defined by the subject-matter experts of law enforcement officials.

#### 3.2. The notation

**Table2.** Notation used for the models

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Sets	
$G$	The directed and capacitated graph
$N$	Set of all nodes in network $G$ .
$V$	Set of middle nodes in network $G$ .
$A$	Set of all arcs in network $G$ .
$O$	Set of origins in network $G$ .
$D$	Set of destinations in network $G$ .
$K$	Set of commodities
$S$	Set of scenarios.
Indices	
$i, j \in N$	Index used for nodes of network $G$ .
$k \in K$	Index used for type of commodities in the network.
$s \in S$	Index used for scenario.
Parameters	
$u_{ij}$	Capacity of arc $(i, j)$
$p_s$	The occurrence probability of scenario $s$
$h_{ij}$	The required cost of interdicator for interdiction in arc $(i, j)$
$Bp$	Total budget available to police
$BS$	Total budget available to smugglers
$\hat{x}_{ij}$	The operator's estimate for $x_{ij}$ which is a function of $x_{ij}$ , $x_{0ij}$ and $c_{ij}$
$cm_{ij}$	The cost for which the amount of interdiction applied to the arc $(i, j)$ is correctly estimated
$x_{0ij}^s$	The initial smuggler's estimate of the amount of interdiction in the arc $(i, j)$ carried out without any expense under scenario $s$
Decision Variables	
$x_{ij}$	If the arc $(i, j)$ is selected by the police to be interdicted, takes 1, and zero, otherwise
$y_{ijk}^s$	The flow rate of commodity $k$ on arc $(i, j)$ under scenario $s$
$c_{ij}^s$	The spent cost by smuggler to nose around the interdiction in the arc $(i, j)$ (i.e., the cost to estimate $x_{ij}$ ) under scenario $s$

### 3.3. Discrete Maximum Flow Network Interdiction with Asymmetric Information (DMFNIAI)

The investigated problem in this section is formulated as a two-stage bi-level mixed-integer programming with a mini-max objective function. In this model, the interdiction variables are set to zero and one. When

an arc is interdicted ( $x_{ij} = 1$ ), it is completely eliminated from the network. The smuggler has an initial estimate of the interdiction actions on arcs ( $x_{0ij}$ ) which can take only two values of zero or one. By spending money ( $c_{ij}$ ), smugglers can learn about the practice of interdiction ( $\hat{x}_{ij}$ ) to improve the initial estimation. The smugglers' estimate of interdiction is formulated as follows:

$$\hat{x}_{ij} = c_{ij} \cdot x_{ij} + (1 - c_{ij})x_{0ij} \quad (1)$$

Where,  $c_{ij}$  is a binary variable; if the smuggler spends to nose around the interdiction in the arc  $(i, j)$ , its value is one, and we have  $\hat{x}_{ij} = x_{ij}$ , If the smuggler doesn't spend to nose around the interdiction ( $c_{ij} = 0$ ), he relies on his initial estimation ( $\hat{x}_{ij} = x_{0ij}$ ).

On the other hand, the police are unaware of the smuggler's initial estimate ( $x_{0ij}$ ). They do not know on which arcs the smuggler may investigate and nose around the police actions ( $c_{ij}^s$ ). Therefore, the parameter  $x_{0ij}$  is defined as an uncertain parameter ( $x_{0ij}^s$ ) and variable  $c_{ij}$  is assumed as a stochastic variable ( $c_{ij}^s$ ) and defined under different discrete scenarios ( $s$ ). The occurrence probability of scenario  $s$  is denoted by  $p_s$ . The scenarios and associated probabilities are assumed to be predefined by the law enforcement subject-matter expert or can be estimated based on the historical data. Since  $x_{0ij}^s$  is a binary parameter, so the probability density function is a binomial distribution function with probability success  $p$ , which is set to 0.5 in experimental result section.

Taking uncertainty conditions into account, DMFNIAI is modeled by the two-stage stochastic programming approach. The uncertain parameter  $x_{0ij}^s$  is pre-defined by a set of discrete scenarios. The upper-level variables of the problem are design variables (police actions) which are not subject to uncertainty ( $x_{ij}$ ), while the lower-level problem variables are control ones (smuggler reactions) and may take different values under the different scenarios ( $y_{ijk}^s, c_{ij}^s$ ). The objective function is calculated as the expected value of total flow of smuggling goods under different scenarios. According to the above description, the mathematical model is formulated as follows:

$$\text{Minimize } E[f^s(x)] \quad (2)$$

s.t.

$$\sum_{i,j} h_{ij} x_{ij} \leq Bp \quad (3)$$

$$f^s(x) = \text{Maximize } E\left[\sum_{i \in N, j \in D, k \in K, s \in S} y_{ijk}^s\right] \quad (4)$$

$$\sum_i y_{ijk}^s - \sum_i y_{jik}^s = 0 \quad \forall j \in V, \forall k \in K, \forall s \in S \quad (5)$$

$$\sum_{k \in K} y_{ijk}^s \leq u_{ij} (1 - \max\{x_{ij}, x_{ij} \cdot c_{ij}^s + (1 - c_{ij}^s) x_{0ij}^s\}) \quad \forall (i, j) \in A \quad (6)$$

$$\sum_{ij} c_{ij}^s \cdot cm_{ij} \leq Bs \quad \forall s \in S \quad (7)$$

$$x_{ij}, c_{ij}^s \in \{0, 1\} \quad \forall (i, j) \in A, \forall s \in S \quad (8)$$

$$y_{ijk}^s \geq 0 \quad \forall (i, j) \in A, \forall k \in K, \forall s \in S \quad (9)$$

Eq. (2) is the interdicator's objective function, which tries to disrupt network arcs in such a way that they minimize the maximum flow of smuggling commodities under all scenarios. The function  $f^s(x)$  denotes the maximum goods trafficked under scenario  $s$ .

Constraint set (3) indicates the limitations of the police budget. In this limitation, the total spent money by the police should be less than the available budget. Eq. (4) denotes the objective function of the smuggler that attempts to maximize the smuggling goods' flow. Eq. (5) indicates the flow equilibrium in each middle node of the network for each type of commodity and under each scenario. Constraints (6) limit the flow of arc  $(i, j)$  which is determined based on the capacity of the arc, the police network interdiction and smuggler's reactions. In other words, the smuggling flow in arc  $(i, j)$  under each scenario depends on the police interdiction  $(x_{ij})$ , the initial estimate of smuggler  $(x_{0ij}^s)$  and his decision to nose around the arc by spend money  $(c_{ij}^s)$ . Constraints (7) indicate the limitation of the smugglers' budget under different scenarios. Constraints (8) and (9) define the variable types.

An endeavor is made to linearize the above-described mathematical model in Appendix A (Nandi AK and Medal HR, 2016).

### 3.4. Continuous Maximum Flow Network Interdiction with Asymmetric Information (CMFNIAI)

In this case, CMFNIAI is formulated as a two-stage bi-level mixed-integer programming with a mini-max objective function. However, the difference is that the police can have a partial interdiction and the arc will not be deleted if the interdiction is not complete.

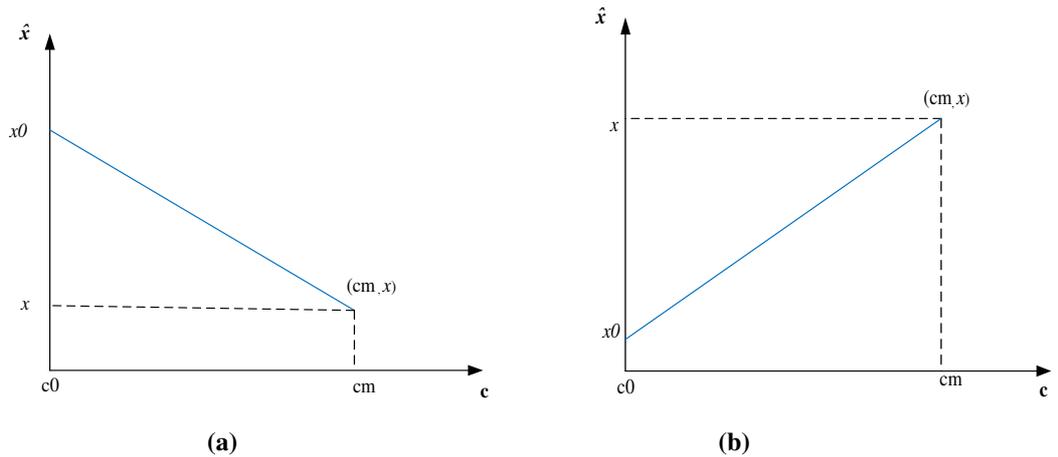
In this model, the applied interdiction has been made according to the construction of checkpoints  $(x_{ij})$ . If the police set up the maximum allowable number of checkpoints  $(UX_{ij})$  in arc  $(i, j)$ , it is possible to have complete interdiction. Furthermore, the smuggler is not completely aware of the imposed interdiction situation. In fact, the smuggler has an initial estimate of the number and location of checkpoints, but he can spend money to improve his initial estimation by for instance bribery, hacking, hiring local informant, and/or unauthorized access to police database, etc., (each one obviously has its own cost). Using a two-

stage stochastic programming approach, the problem is formulated under the nondeterministic condition. The parameter  $x_{0ij}^s$  is assumed random, and all lower-level variables are subject to uncertainty ( $y_{ijk}^s, c_{ij}^s$ ). When the actual number ( $x_{ij}$ ) or the estimated number ( $\hat{x}_{ij}^s$ ) of checkpoints increases, the smuggling flow on that arc decreases. In other words, when  $x_{ij}$  increases, it will result in partial interdiction of arc  $(i, j)$  and implies that the police confiscate the smuggling flow of that arc accordingly, and when  $\hat{x}_{ij}^s$  increases, it will make the smuggler conscious and implies that they reduce the density of smuggling good in this arc, and probably try other alternative arcs. Therefore, in both cases, the smuggling goods' flow on that arc is reduced. That is why we use a maximum operator to describe this situation (see Constraint (12)).

As previously discussed, the asymmetric information of smuggler toward the police checkpoints is formulated by a linear function. In fact, the more money the smugglers spend, the better they will estimate the number of checkpoints. Due to the asymmetric information concept, police are unaware of the smuggler's reaction, so we introduce two types of linear functions to simulate the risk seeking and risk averse smuggler as follows (see Fig. 1):

**Overestimate:** As its name implies, the initial estimate of the number of police checkpoints is always higher than its actual number. This state of affairs represents a risk-averse situation. from the smuggler point of view.

**Underestimate:** In this case, the initial estimate of the number of police checkpoints is always lower than its actual number. This state of affairs represents a risk-seeking situation from the smuggler point of view.



**Fig. 1.** (a) Overestimate, (b) Underestimate

According to Figure 1, the two points  $(0, x_0)$  and  $(cm, x)$  have the following linear relationship:

$$\hat{x} - x_0 = \frac{x - x_0}{cm - 0}(c - 0) \quad \Rightarrow \quad \hat{x} = x_0 + \frac{x - x_0}{cm}c \quad (c < cm) \quad (10)$$

Where  $\hat{x}$  is the number of checkpoints estimated by the smuggler,  $x$  is the actual number of checkpoints,  $c$  is the spent amounts of money by the smuggler to estimate the number of checkpoints and  $cm$  is the cost needed to estimate the actual number of checkpoints. The CMFIAI model is therefore presented as follows:

$$\text{Minimize } E[f^s(x)] \quad (11)$$

s.t.

$$\sum_{k \in K} y_{ijk}^s \leq u_{ij} \left( 1 - \max \left( \frac{x_{ij}}{UX_{ij}}, \frac{\hat{x}_{ij}^s}{UX_{ij}} \right) \right) \quad \forall (i,j) \in A, \forall s \in S \quad (12)$$

$$\sum_{ij} c_{ij}^s \leq Bs \quad \forall s \in S \quad (13)$$

$$0 \leq c_{ij}^s \leq cm_{ij} \quad \forall (i,j) \in A, \forall s \in S \quad (14)$$

$$0 \leq x_{ij} \leq UX_{ij} \quad \forall (i,j) \in A \quad (15)$$

$$x_{ij} \in Z^+ \quad \forall (i,j) \in A \quad (16)$$

$$y_{ijk}^s, c_{ij}^s \geq 0 \quad \forall (i,j) \in A, \forall k \in K, \forall s \in S \quad (17)$$

, and Constraints 3-5

Where,  $x_{ij}$  is the number of checkpoints set up by the police in arc  $(i, j)$ .

Eq. (11) presents the interdicator's objective function, which tries to disrupt network arcs in such a way that they minimize the maximum amount of smuggling goods flows under all scenarios. Constraints (12) limit the flow of arc  $(i, j)$ . In fact, each arc has an initial capacity of  $u_{ij}$ , and this capacity can be limited according to the construction of the stopping checkpoints. The right hand side is a function of the arc's initial capacity, the number of actual and estimated checkpoints. The actual number of checkpoints in fact reflects the action of the police and implies the confiscation of smuggling goods, while the estimated number of checkpoints is the reaction of the smuggler, and implies that the smuggler refuses to cross that arc and probably utilize the alternative arcs. Constraint (13) denotes the smuggler's budget limitation. In fact, the cost spent by a smuggler to estimate the number of police checkpoints over the network should be less than his available budget. Constraint (14) restricts the cost required to estimate the actual number of checkpoints with an upper bound. Constraint (15) specifies the allowed number of checkpoints can be set up in each arc. Constraints (16) and (17) define the variable types.

It is assumed that the estimated number of checkpoints has a linear relationship with the cost spent by the smuggler. Considering Eq. (10), we have:

$$\hat{x}_{ij}^s = x_{0ij}^s + \frac{x_{ij} - x_{0ij}^s}{cm_{ij}} c_{ij}^s \quad (18)$$

By inserting the Eq. (18) in the above model, we have:

$$\text{Minimize } f^s(x) \quad (19)$$

s.t.

$$\sum_{k \in K} y_{ijk}^s \leq u_{ij} \left(1 - \left(\frac{x_{ij}}{UX_{ij}}\right)\right) \quad \forall (i,j) \in A, \forall s \in S \quad (20)$$

$$\sum_{k \in K} y_{ijk}^s \leq u_{ij} \left(1 - \left(\frac{x_{0ij}^s}{UX_{ij}} + \frac{x_{ij} \cdot c_{ij}^s}{UX_{ij} \cdot cm_{ij}} - \frac{x_{0ij}^s}{UX_{ij} \cdot cm_{ij}} c_{ij}^s\right)\right) \quad \forall (i,j) \in A, \forall s \in S \quad (21)$$

, and Constraints (3)-(5), (13)-(17)

Due to the multiplication of two discrete and continuous variables in Constraint (21), CMFNIAI is nonlinear in its present form and should be linearized. The steps of linearization are presented in Appendix B.

The equivalent linearized CMFNIAI model could be rewritten as follows:

$$\text{Minimize } f^s(x) \quad (22)$$

s.t.

$$\sum_{i,j} h_{ij} \sum_{l=0}^{n-1} 2^l t_{ijk} + (U_{X_{ij}} - 2^n + 1)t_{ijn} \leq Bp \quad (23)$$

$$\sum_{k \in K} y_{ijk}^s \leq u_{ij} \left(1 - \left(\frac{\sum_{l=0}^{n-1} 2^l t_{ijk} + (U_{X_{ij}} - 2^n + 1)t_{ijn}}{U_{X_{ij}}}\right)\right) \quad \forall (i,j) \in A, \forall s \in S \quad (24)$$

$$\sum_{k \in K} y_{ijk}^s \leq u_{ij} \left(1 - \left(\frac{x_{0ij}^s}{U_{X_{ij}}}\right)\right) \quad \forall (i,j) \in A, \forall s \in S \quad (25)$$

$$+ \left(\frac{\sum_{l=0}^{n-1} 2^l z_{ijk}^s + (U_{X_{ij}} - 2^n + 1)z_{ijn}^s}{U_{X_{ij}} \cdot cm_{ij}} - \frac{x_{0ij}^s}{U_{X_{ij}} \cdot cm_{ij}} c_{ij}^s\right)$$

$$c_{ij}^s - (1 - t_{ijk}) \cdot M \leq z_{ijk}^s \quad \forall (i,j) \in A, \forall k \in K, \forall s \in S \quad (26)$$

$$z_{ijk}^s \leq c_{ij}^s \quad \forall (i,j) \in A, \forall k \in K, \forall s \in S \quad (27)$$

$$z_{ijk}^s \leq M \cdot t_{ijk} \quad \forall (i,j) \in A, \forall k \in K, \forall s \in S \quad (28)$$

$$t_{ijk} \in \{0,1\} \quad \forall (i,j) \in A, \forall k \in K \quad (29)$$

$$z_{ijk}^s \geq 0 \quad \forall (i,j) \in A, \forall k \in K, \forall s \in S \quad (30)$$

, and Constraints (4)-(5), (13)-(14), (17)

## 4. Solution Methodology

This section is two-fold, first we introduce a hybrid exact algorithm method to solve the DMFNIAI model, and then an efficient reformulation method is presented to solve the CMFNIAI model.

### 4.1. Solution method for DMFNIAI (DM-PHA)

Solving a two-stage bi-level optimization problem is highly challenging. Owing to the high application value of a bi-level optimization problem, substantial efforts have been undertaken to solve it. In general, the exact solution method mainly includes benders decomposition (Israeli & Wood, 2002; Morton, 2007; Sadeghi et al., 2017), reformulation techniques (Linderoth, 2008), and implicit enumeration (Sarhadi et al., 2017). However, reformulation techniques are not applicable in this work because the lower-level variables in DMFNIAI model are not continuous. Furthermore, neither the implicit enumeration method nor Benders decomposition algorithms cannot be used effectively to solve the proposed model. This is because the lower-level program is a highly complicated nonlinear model (Xiang et al. 2020). In order to solve the DMFNIAI problem (2-9), we propose a hybrid algorithm which is hereunder called the DM-PHA algorithm which is comprising of a two-phase Decomposition Method (DM) and the Progressive Hedging Algorithm (PHA). The proposed decomposition algorithm is inspired from Fischetti et al. (2018), which is a very efficient and effective heuristic and chosen to handle the bi-level structure of the problem. The PHA algorithm is a well-known algorithm and integrated in our solving approach to cope with the two-stage stochastic structure of the proposed model. In fact, the decomposition algorithm is embedded inside the PHA loops to handle the non-convexity of the bi-level structure of the model. The Optimization Programming Language (OPL) and Flow Control Script accessed via *IBM ILOG CPLEX 12.8* have been used to Code the DM-PHA algorithm.

#### 4.1.1. Progressive Hedging Algorithm (PHA)

To explain the proposed hybrid algorithm, one should first describe the PHA algorithm. PHA algorithm is a scenario-based decomposition technique that can be leveraged to solve multi-stage stochastic programs. According to Watson and Woodruff (2011), although PHA is originally devised for problems with purely continuous variables, it has been successfully applied as a heuristic to solve multi-stage stochastic programs with integer variables. In order to show the PHA steps, a classic formulation of the two-stage stochastic linear program is used as follows (Sahinidis, 2004, Mirzapour Al-e-hashem, et al., 2019):

$$\begin{aligned} \min c^t x + E_{\omega \in \Omega}[Q(x, \omega)] \quad & x \in X \\ \text{s.t.} \end{aligned} \tag{31}$$

$$Q(x, \omega) = \min f(\omega)^t y \quad (32)$$

Where  $Q(x, \omega)$  is the optimal value of the second stage problem:

$$D(\omega)y \geq h(\omega) + T(\omega)x \quad y \in Y \quad (33)$$

Where  $x$  and  $y$  are vectors of the first and second stage decision variables, respectively. The second stage problem depends on the data  $\Omega = (f, D, h, T)$  where any or all elements can be a function of random variable  $\omega$ . Eq. (31) with variables  $x$  constitute the first stage which need to be decided prior to the realization of the uncertain parameters  $\omega \in \Omega$ . Eq. (32), (33) with variables  $y$  constitute the second stage and are subject to uncertainty. Under the two-stage setting, the first stage variables are not dependent to the probability of the scenarios. By breaking this assumption, we suppose that the variables of the first stage ( $x$ ) depend on the probability of the scenarios. In such a case, the model can be converted to a single-stage formulation as follows (Gade et al. 2016):

$$\text{Min} \sum_{\omega} P [c^T x(\omega) + f(\omega)^T y(\omega)] \quad (34)$$

s.t.

$$Dy(\omega) \geq h(\omega) - T(\omega)x(\omega) \quad (35)$$

$$P_{\omega} x(\omega) - P_{\omega} \hat{x} = 0 \quad (36)$$

Eq. (36) is called non-anticipativity constraint and added to retain the independence of the first stage decision variables from the scenarios, and makes the model (34-36) equivalent to its main form (31-33).

Given the above mentioned model (34-35) (Eq. (36) is temporary excluded from the model but it will be guaranteed by the algorithm using a penalty function), the statement of the PHA for two-stage stochastic mixed-integer programs (SMIP) can be summarized as follows (Watson et al., 2011; Gade et al., 2016):

**Table3.** PHA Algorithm (Watson et al., 2011)

1. Initialization: Let  $v \leftarrow 0$  and  $w(\omega) \leftarrow 0, \forall \omega \in \Omega$ . For each  $\omega \in \Omega$ , compute:

$$(x^{v+1}(\omega), y^{v+1}(\omega)) \in \text{argmin } c^T x + f(\omega)^T y$$

2. Iteration Update:  $v \leftarrow v + 1$

3. Aggregation:  $\hat{x}^v \leftarrow \sum_{\omega} P_{\omega} x^v(\omega)$

4. Price Update:  $w^v(\omega) \leftarrow w^{v-1}(\omega) + \rho(x^v(\omega) - \hat{x}^v)$

5. Decomposition: For each  $\omega \in \Omega$ , compute:

$$(x^{v+1}(\omega), y^{v+1}(\omega)) \in \text{argmin } c^T x + f(\omega)^T y + w^v(\omega)^T x + \frac{\rho}{2} \|x - \hat{x}^v\|^2$$

6. If all scenario solutions  $x(\omega)$  are equal, stop. Else, go to Step 2.

The PHA is initialized by solving the individual scenario problems (Step 1). Each iteration of the PHA involves an *aggregation* operation (Step 3), which corresponds to a projection of the individual scenario solutions onto the subspace of non-anticipative policies. The dual prices  $w^v(\omega)$  are then updated (Step 4), using the sole external parameter associated with the basic PHA:  $\rho$ . The decomposition step of each PHA iteration (Step 5) involves solving scenario problems whose first-stage costs have been perturbed by the dual prices. Further, the objective function in this step is modified to include a proximal term that measures the deviation of the scenario solution from the aggregated first-stage policy  $\hat{x}^v$  using the squared two-norm. In practical applications, the test for convergence in Step 6 of the algorithm requires only convergence to within a tolerance for non-integer variables (Gade et al., 2016). Therefore, in PHA algorithm, we solve the problem for each scenario separately, in fact the main problem is decomposed to n sub-problems each of which is solved under a different scenario dataset. But, as previously discussed, the obtained single-stage deterministic sub-problem is a bi-level model, which should be converted to a single-stage before it can be solved. So, in each iteration of the PHA, and for each scenario, the decomposition algorithm is called to solve the bi-level sub-problem.

#### 4.1.2. Decomposition Method (DM)

As described above, the PHA algorithm is appropriate to solve a two-stage single-level problems. Therefore, in order to make use of PHA, we must convert the primary two-stage bi-level model to a single level equivalent one. Thereby, we use a two-phase decomposition method introduced by Fischetti et al., (2018), which their steps are given in Tables 4 and 5, respectively.

**Table 4.** Decomposition Method; Phase\_1 (One-Shot Algorithm)

**Input:** The DMFNIAI model (2) -(9);

**Output:** A heuristic DMFNIAI solution  $(\bar{x}, \bar{c})$ ;

1. Relax the integrality of the  $c_{ij}$  variables, namely constraint (8);
2. Reformulate the problem as  $\overline{\text{DMFNIAI}}$ ;
3. Solve the resulting single-level MILP and let  $(\bar{x}, 0)$  be the optimal (or best) solution found;
4. Refine  $(\bar{x})$  (Phase\_2);
5. return  $(\bar{x}, \bar{c})$ ;

#### The single-level form of the MFNIAI problem ( $\overline{\text{DMFNIAI}}$ )

Phase\_1 of the decomposition method (One-Shot) is used to find a reasonable leader vector, say  $\hat{x}_{ij}$ , to feed its phase\_2. This is obtained by relaxing the integrality of the  $c_{ij}$  variables into the smuggler problem (inner level) (See Appendix C) . According to the following notations, the dual of the above-described problem is given in Appendix D:

- $\pi_{jk}$       The dual variable corresponding to Equation (5)
- $\Omega_{ij}$       The dual variable corresponding to Equation (C.2)
- $\theta$           The dual variable corresponding to Equation (7)

Given the dual model of the lower level problem (See Appendix D), and with the help of the weak duality theorem, the single level model ( $\overline{\text{DMFNIAI}}$ ) is then formulated as follows:

$$\text{Min} \sum_{i \in N, j \in D, k \in K} y_{ijk} \quad (37)$$

s.t.

$$\sum_{k \in K} y_{ijk} \leq u_{ij}(1 - x_{ij}) \quad \forall (i, j) \in A \quad (38)$$

$$\sum_{k \in K} y_{ijk} \leq u_{ij}(1 - (x_{ij} \cdot c_{ij} + (1 - c_{ij})x_{0ij})) \quad \forall (i, j) \in A \quad (39)$$

$$u_{ij} \cdot \Omega_{ij} + Bs. \theta \geq \sum_{i \in N, j \in D, k \in K} y_{ijk} \quad \forall (i, j) \in A \quad (40)$$

$$y_{ijk}, \Omega_{ij}, \theta, c_{ij} \geq 0 \quad \forall (i, j) \in A, \forall k \in K \quad (41)$$

$$x_{ij} \in \{0, 1\} \quad \forall (i, j) \in A \quad (42)$$

$$\pi_{jk}: \text{unrestricted in sign variables} \quad \forall j \in V, \forall k \in K \quad (43)$$

, and Constraints 3, 5, 7, D.1-D.5

As Table 4 shows, the step 4 in the first phase (One-Shot algorithm) of the decomposition method recalls the phase\_2 (named as Refine Algorithm) which its steps are described in Table 5:

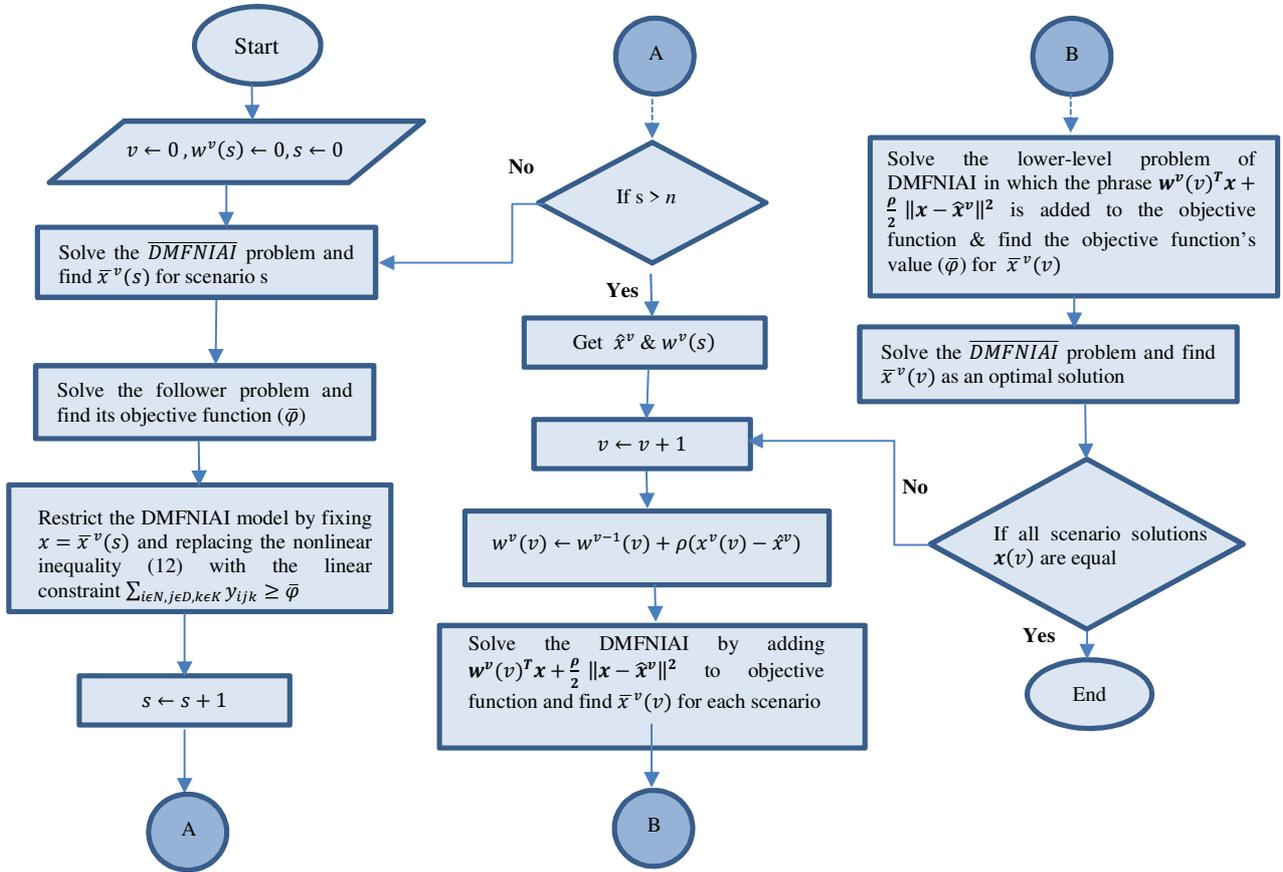
**Table 5.** Decomposition Method; Phase\_2 (Refine Algorithm)

<p><b>Input:</b> A leader solution <math>\hat{x}</math>;</p> <p><b>Output:</b> A heuristic DMFNIAI solution <math>(\bar{x}, \bar{c})</math>;</p> <ol style="list-style-type: none"> <li>1. Solve the follower MILP for <math>x=\bar{x}</math> to compute <math>\bar{\varphi} = \Phi(\bar{x})</math>;</li> <li>2. Restrict the DMFNIAI model by fixing <math>x = \bar{x}</math> and replacing <math>\sum_{i \in N, j \in D, k \in K} y_{ijk} \geq \bar{\varphi}</math> with the nonlinear expression, the objective function of the inner problem;</li> <li>3. Solve the resulting MILP;</li> <li>4. return <math>(\bar{x}, \bar{y})</math>;</li> </ol>
--

Refine Algorithm specifies the refinement process with which the initial heuristic solution of DMFNIAI in phase\_1 is goes to the upper level and creates a high quality solution for the bi-level problem. The presented decomposition method typically requires little computational time, because both phases are much simpler

than the main nonlinear DMFNIAI model. It is worth mentioning that solution  $(\bar{x}, \bar{c})$  obtained from phase\_2, in addition to being feasible, is a higher quality solution than its input solution (phase\_1). That is why this correspondence is called the refining procedure.

The computational complexity of the decomposition method has been investigated in Caprara (2014) and complexity of the PHA algorithm is demonstrated as  $(N^2)$  (Lu et al. (2013)), and they collectively make sure the complexity of the proposed algorithm in worse-case is polynomial. In order to better understanding of the proposed hybrid algorithm (DM-PHA), the entire process of applying this algorithm is presented in a flowchart (see Fig. 2).



**Figure 2.** Flow chart of the proposed hybrid algorithm (DM-PHA)

#### 4.2. Solution method for CMFNIAI (Reformulation)

In this section, we propose two solution methods for the CMFNIAI problem presented in Equations (22) - (30). Since the lower level variables are all continuous, therefore the dual of the lower level problem is definable and consequently the bi-level problem can be converted to a single level model. Hence, to solve this problem a classic reformulation technique based on Karush-Kuhn-Tucker (KKT) conditions is first proposed and then compared with an augmented reformulation technique.

The lower level problem (smuggler problem) of CMFNIAI could be rewritten as follows:

$$\text{Maximize} \left( \sum_{i \in N, j \in D, k \in K, s \in S} y_{ijk}^s \right) \quad (44)$$

s.t.

, and Constraints 5, 13-14, 17, 20-21

According to the following notations, the dual of the above problem can be written as described in Appendix E.

- $\pi_{jk}^s$       The dual variable corresponding to Constraint (5)
- $\Omega 1_{ij}^s$     The dual variable corresponding to Constraint (20)
- $\Omega 2_{ij}^s$     The dual variable corresponding to Constraint (21)
- $\theta 1^s$       The dual variable corresponding to Constraint (13)
- $\theta 2_{ij}^s$     The dual variable corresponding to Constraint (14)

By adding the dual constraints of the inner problem as well as the complementary-slackness condition, the CMFNIAI can be converted to a single-level equivalent problem as follows:

### Single-level CMFNIAI Problem

$$\text{Minimize} \left( \sum_{i \in N, j \in D, k \in K, s \in S} y_{ijk}^s \cdot p_s \right) \quad (45)$$

s.t.

$$\pi_{jk}^s - \pi_{ik}^s + \Omega 1_{ij}^s + \Omega 2_{ij}^s \geq 1 \quad \forall (i, j) \in A, \forall k \in K, \forall s \in S \quad (46)$$

$$-\pi_{jk}^s + \Omega 1_{ij}^s + \Omega 2_{ij}^s \geq 1 \quad \forall i \in O, \forall j \in V, \forall k \in K, \forall s \in S \quad (47)$$

$$\pi_{ik}^s + \Omega 1_{ij}^s + \Omega 2_{ij}^s \geq 1 \quad \forall j \in D, \forall i \in V, \forall k \in K, \forall s \in S \quad (48)$$

$$\frac{x_{ij} \cdot u_{ij}}{cm_{ij} \cdot U_{X_{ij}}} \cdot \Omega 2_{ij}^s - \frac{x_{0ij} \cdot u_{ij}}{cm_{ij} \cdot U_{X_{ij}}} \cdot \Omega 2_{ij}^s + \theta 1 + \theta 2_{ij} \geq 0 \quad \forall i, j \in N, \forall k \in K, \forall s \in S \quad (49)$$

$$\left( u_{ij} \left( 1 - \frac{x_{ij}}{U_{X_{ij}}} \right) - \sum_{k \in K} y_{ijk}^s \right) \leq M v_{1ij}^s \quad \forall (i, j) \in A, \forall k \in K, \forall s \in S \quad (50)$$

$$\Omega 1_{ij}^s \leq M(1 - v_{1ij}^s) \quad \forall (i, j) \in A, \forall s \in S \quad (51)$$

$$u_{ij} \left( 1 - \left( \frac{x_{0ij}^s}{UX_{ij}} + \frac{x_{ij} \cdot c_{ij}^s}{UX_{ij} \cdot cm_{ij}} - \frac{x_{0ij}^s \cdot c_{ij}^s}{UX_{ij} \cdot cm_{ij}} \right) \right) - \sum_{k \in K} y_{ijk}^s \leq Mv_{2ij}^s \quad \forall (i, j) \in A, \forall k \in K, \forall s \in S \quad (52)$$

$$\Omega 2_{ij}^s \leq M(1 - v_{2ij}^s) \quad \forall (i, j) \in A, \forall s \in S \quad (53)$$

$$B_s - \sum_{i,j} c_{ij}^s \leq Mv_3 \quad (54)$$

$$\theta 1 \leq M(1 - v_3) \quad (55)$$

$$cm_{ij} - c_{ij}^s \leq Mv_{4ij} \quad \forall (i, j) \in A \quad (56)$$

$$\theta 2_{ij} \leq M(1 - v_{4ij}) \quad \forall (i, j) \in A \quad (57)$$

$$\pi_{jk}^s - \pi_{ik}^s + \Omega 1_{ij}^s + \Omega 2_{ij}^s - 1 \leq Mw_{1ijk}^s \quad \forall (i, j) \in A, \forall k \in K, \forall s \in S \quad (58)$$

$$y_{ijk}^s \leq M(1 - w_{1ijk}^s) \quad (59)$$

$$-\pi_{jk}^s + \Omega 1_{ij}^s + \Omega 2_{ij}^s - 1 \leq Mw_{2ijk}^s \quad \forall i \in O, \forall j \in V, \forall k \in K, \forall s \in S \quad (60)$$

$$y_{ijk}^s \leq M(1 - w_{2ijk}^s) \quad (61)$$

$$\pi_{ik}^s + \Omega 1_{ij}^s + \Omega 2_{ij}^s - 1 \leq Mw_{3ijk}^s \quad \forall j \in D, \forall i \in V, \forall k \in K, \forall s \in S \quad (62)$$

$$y_{ijk}^s \leq M(1 - w_{3ijk}^s) \quad (63)$$

$$\frac{x_{ij} \cdot u_{ij}}{cm_{ij} \cdot UX_{ij}} \cdot \Omega 2_{ij}^s - \frac{x_{0ij}^s \cdot u_{ij}}{cm_{ij} \cdot UX_{ij}} \cdot \Omega 2_{ij}^s + \theta 1 + \theta 2_{ij} \leq Mw_{4ij}^s \quad (64)$$

$$cij \leq M(1 - w_{4ijk}^s) \quad (65)$$

$$y_{ijk}^s, c_{ij}^s, \Omega 1_{ij}^s, \Omega 2_{ij}^s, \theta 1, \theta 2_{ij} \geq 0, x_{ij} \in Z \quad (66)$$

$$v_{1ij}^s, v_{2ij}^s, v_3, v_{4ij}, w_{1ijk}^s, w_{2ijk}^s, w_{3ijk}^s, w_{4ij}^s \in \{0, 1\}, \pi_{jk}: urs \quad \forall (i, j) \in A, \forall k \in K, \forall s \in S \quad (67)$$

, and Constraints 3, 5, 13-15, 20-21

### Augmented Reformulation for CMFNIAI Problem

In order to solve the CMFNIAI more efficiently, a new reformulation is presented, which is much more efficient than the regular reformulation based on KKT conditions (Garcés et al., 2009), because the number of constraints and the number of variables are fewer. In this method, by adding a new constraint (strong duality theorem) which guarantees the equality of the objective functions of the primal and dual problems, the necessity for complimentary constraints is resolved and therefore can be omitted. In addition, by eliminating the objective function of the inner problem, the model will be single level and the problem can be optimally solved. The equivalent single-level model is therefore can be rewritten as follows:

$$\text{Minimize} \left( \sum_{i \in N, j \in D, k \in K, s \in S} y_{ijk}^s \right) \quad (68)$$

s.t.

$$u_{ij} \left( 1 - \left( \frac{x_{ij}}{UX_{ij}} \right) \right) \Omega 1_{ij}^s + u_{ij} \left( 1 - \left( \frac{x_{0ij}^s}{UX_{ij}} \right) \right) \Omega 2_{ij}^s + Bs\theta 1 + cm_{ij} \cdot \theta 2_{ij} \quad \forall (i, j) \in A, \forall s \in S \quad (69)$$

$$= \left( \sum_{i \in N, j \in D, k \in K, s \in S} y_{ijk}^s \right)$$

$$y_{ijk}^s \cdot c_{ij}^s \cdot \Omega 1_{ij}^s \cdot \Omega 2_{ij}^s \cdot \theta 1 \cdot \theta 2_{ij} \geq 0, \quad x_{ij} \in Z, \pi_{jk}: \text{urs} \quad \forall (i, j) \in A, \forall k \in K, \forall s \in S \quad (70)$$

, and Constraints 3, 5, 13-15, 20-21, 46-49

Equations (46) – (49) express the dual constraints of the inner problem, and Eq. (69) states the strong duality theorem.

## 5. Computational experiments

In this section, we first describe a test problem and solve it for discrete and continuous setting by using the DMFNIAI and CMFNIAI, respectively. The aim of this section is to show the applicability of the proposed models as well as the developed algorithms. Then we will perform some sensitivity analysis to demonstrate how the proposed models can provide high-quality solutions in order to interdict the smuggling goods, under information asymmetry conditions.

For this purpose, a network of 18 nodes is investigated including 6 origins, 6 destinations and 6 middle nodes which are shown by the triangle, circle, and rectangle respectively, in Fig. 3.

### 5.1. Test problem for DMFNIAI

In this example, we consider two scenarios, each with an occurrence probability of 0.5, meaning that each parameter  $x_{0ij}$  is randomly generated. If  $x_{0ij}^1 = 1$ , it is a risk-averse, and if  $x_{0ij}^2 = 0$ , signifies a risk-seeking smuggler. Given that  $Bp = 500$  and  $Bs = 1000$ , DMFNIAI model is optimally solved. The optimal value of the objective function is 1610 and the other obtained result is reported in Table 6.

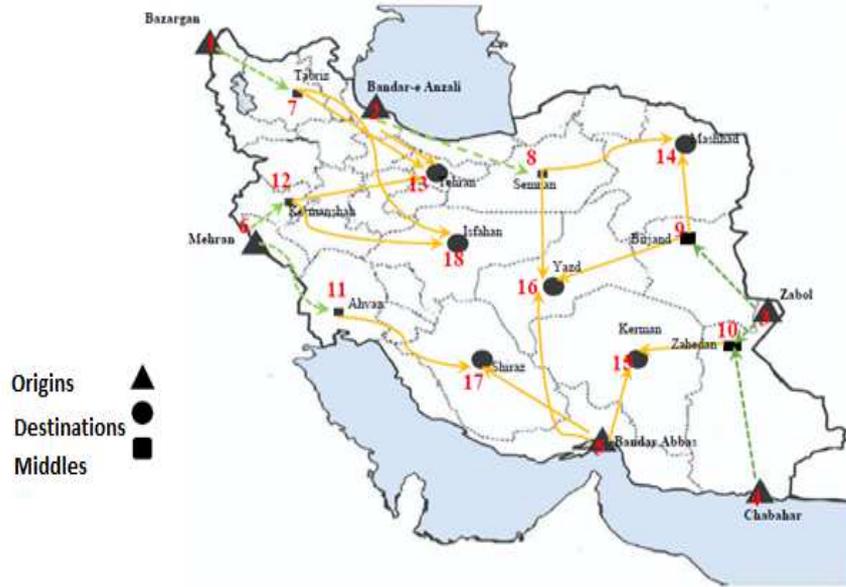


Fig. 3. Schematic network of the test problem

Table 6. The obtained results of DMFNIAI problem with  $Bp = 500$ , and  $Bs = 1000$ .

(i, j)	$x_{ij}$	$C_{ij}^s$		$\hat{x}_{ij}^s$		$\mathcal{Y}_{ijk}^s$					
		s=1	s=2	s=1	s=2	k=1		k=2		k=3	
						s=1	s=2	s=1	s=2	s=1	s=2
(1,7)	0	1	1	0	0	70	70	70	70	60	60
(2,8)	0	0	0	1	0	0	0	0	0	0	0
(2,13)	1	1	0	1	0	0	0	0	0	0	0
(3,9)	0	0	0	1	0	0	0	0	0	0	0
(3,10)	1	1	1	1	1	0	0	0	0	0	0
(4,10)	0	1	0	0	0	30	30	35	35	0	0
(5,15)	1	0	0	1	0	0	0	0	0	0	0
(5,16)	0	1	1	0	0	50	50	35	35	10	10
(5,17)	0	0	1	1	0	0	0	0	0	0	0
(6,11)	0	0	0	1	0	0	0	0	0	0	0
(6,12)	0	1	1	0	0	50	50	0	0	50	50
(7,13)	1	0	1	1	1	0	0	0	0	0	0
(7,18)	0	1	1	0	0	80	80	70	70	50	50
(8,14)	0	1	0	0	0	0	0	0	0	0	0
(8,16)	0	1	1	0	0	0	0	0	0	0	0
(9,14)	0	0	0	1	0	0	0	0	0	0	0
(9,16)	0	0	0	1	0	0	0	0	0	0	0
(10,15)	0	1	1	0	0	30	30	35	35	0	0
(11,17)	0	0	1	1	0	0	0	0	0	0	0
(12,13)	0	0	0	1	0	0	0	0	0	0	0
(12,18)	0	1	0	0	0	50	50	0	0	50	50

5.2. Test problem for CMFNIAI

In order to examine the CMFNIAI model in the network of Figure 2, a test problem is constructed and input data are given in Table 7, including the arc's capacity ( $u_{ij}$ ), the cost of constructing checkpoints ( $h_{ij}$ ), the maximum amount of investment per arc ( $cm_{ij}$ ) and the maximum number of checkpoints that can be constructed in each arc ( $UX_{ij}$ ). The occurrence probability of the first scenario (optimistic) is assumed 0.3, where  $x_{0ij}^1 = 0$ , and the smuggler is risk-seeking, the occurrence probability of the second scenario (mostly expected) is set to 0.4 with  $x_{0ij}^2 = 5$ , and finally, the third scenario (pessimistic) is defined by the occurrence probability of 0.3 with  $x_{0ij}^3 = 10$  and a risk-averse smuggler.

**Table 7.** Test problem data for solving the CMFNIAI model

(i,j)	$u_{ij}$	$h_{ij}$	$cm_{ij}$	$UX_{ij}$
(1,7)	200	9	5	7
(2,8)	100	1	7	8
(2,13)	100	10	9	2
(3,9)	150	2	10	5
(3,10)	150	3	2	1
(4,10)	100	5	5	5
(5,15)	100	7	7	3
(5,16)	100	9	1	2
(5,17)	100	8	6	3
(6,11)	100	6	8	3
(6,12)	100	4	9	8
(7,13)	100	2	2	9
(7,18)	100	5	4	10
(8,14)	70	5	6	4
(8,16)	30	2	7	2
(9,14)	75	1	2	4
(9,16)	75	5	1	5
(10,15)	250	4	5	5
(11,17)	100	8	8	6
(12,13)	50	3	9	2
(12,18)	50	7	10	1

In Table 8, the optimal solution for the test problem is presented comprising of the number of checkpoints; the cost spent to estimate the number of checkpoints as well as the number of checkpoints estimated on each arc. In addition, in this example, the value of the objective function is obtained zero.

**Table 8.** The obtained results of CMFNIAI problem

(i,j)	$x_{ij}$	$c_{ij}^s$			$\hat{x}_{ij}^s$			$x_{0ij}^s$		
		s = 1	s = 2	s = 3	s = 1	s = 2	s = 3	s = 1	s = 2	s = 3
(1,7)	7	0	0	5	0	5	7	0	5	10
(2,8)	8	0	0	7	0	5	8	0	5	10
(2,13)	2	0	9	9	0	2	2	0	5	10
(3,9)	0	0	0	10	0	5	0	0	5	10
(3,10)	1	2	2	2	1	1	1	0	5	10
(4,10)	4	0	0	5	0	5	4	0	5	10

$(i,j)$	$x_{ij}$	$c_{ij}^s$			$\hat{x}_{ij}^s$			$x_{0ij}^s$		
		$s = 1$	$s = 2$	$s = 3$	$s = 1$	$s = 2$	$s = 3$	$s = 1$	$s = 2$	$s = 3$
(5,15)	3	0	7	7	0	3	3	0	5	10
(5,16)	2	0	1	1	0	2	2	0	5	10
(5,17)	3	0	6	6	0	3	3	0	5	10
(6,11)	3	0	8	8	0	3	3	0	5	10
(6,12)	0	0	0	1.8	0	5	8	0	5	10
(7,13)	0	0	0	2	0	5	0	0	5	10
(7,18)	0	0	0	0	0	5	10	0	5	10
(8,14)	4	0	6	6	0	4	4	0	5	10
(8,16)	1	0	7	7	0	1	1	0	5	10
(9,14)	0	0	0.4	1.2	0	4	4	0	5	10
(9,16)	0	0	0	0.7	0	5	3	0	5	10
(10,15)	5	1	0	5	0	5	5	0	5	10
(11,17)	10	0	0	8	0	5	6	0	5	10
(12,13)	2	0	9	9	0	2	2	0	5	10
(12,18)	1	0	10	10	0	1	1	0	5	10

We solve the above example once more, allows to compare the two reformulation techniques. The input data are the same as presented in Table 7, and the obtained results are similar to the previous results (Table 8). As expected, the augmented reformulation is more efficient and needs less computational time. Table 9 shows a comparison of these two methods.

**Table 9.** Comparison between two reformulation methods

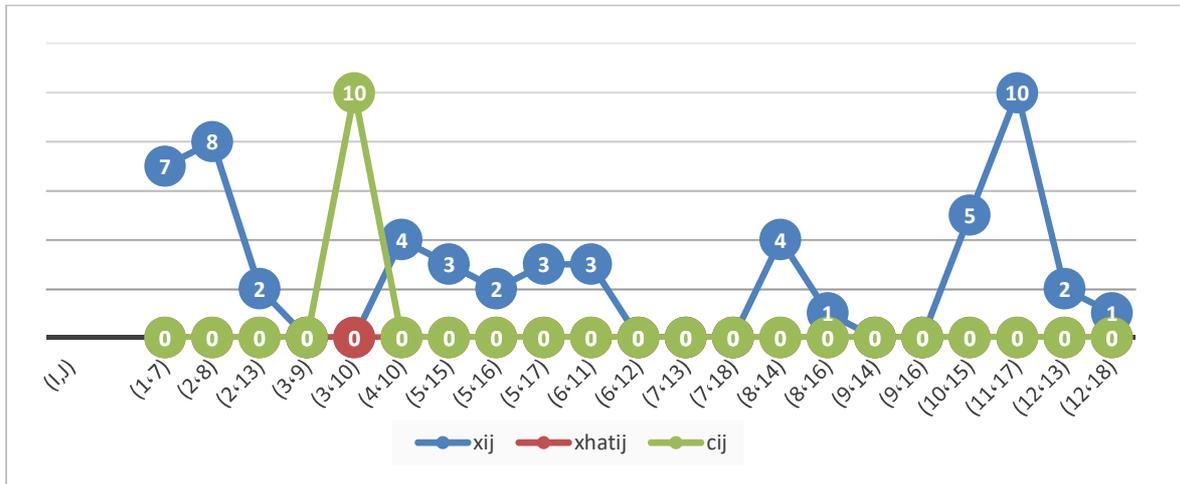
Solution methodology		KKT conditions	Augmented Reformulation
# Constraints		8239	7045
# Variables	Total	3705	3405
	Binary	833	137
	Integer	820	820
Time solving		1.45 min	1.5 sec

### 5.3. The analysis of information asymmetry in proposed models

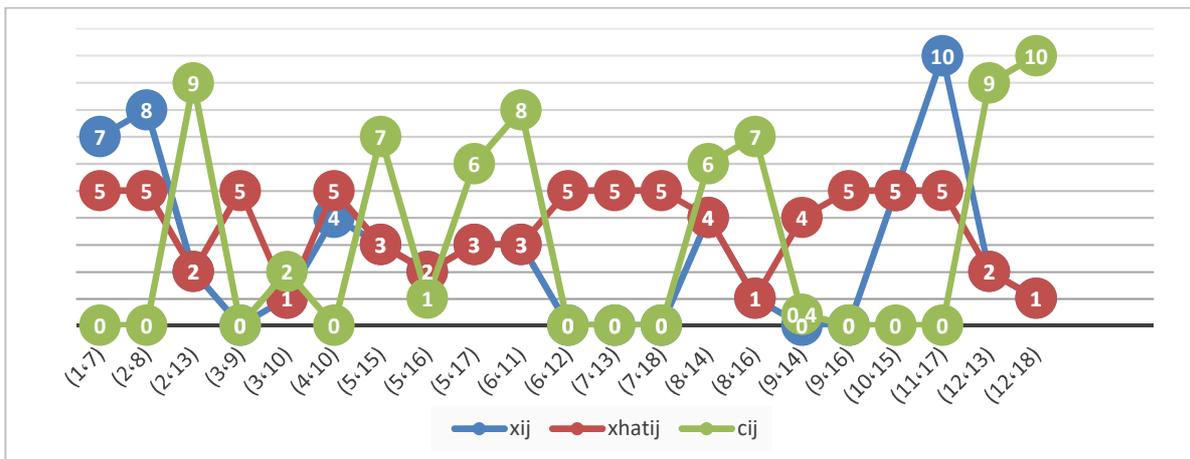
The purpose of this subsection is to describe the efficacy of asymmetric information of the police and smuggler in the maximum flow network interdiction problem for the discrete and continuous setting.

#### 5.3.1. Asymmetric Information on CMFNIAI problem

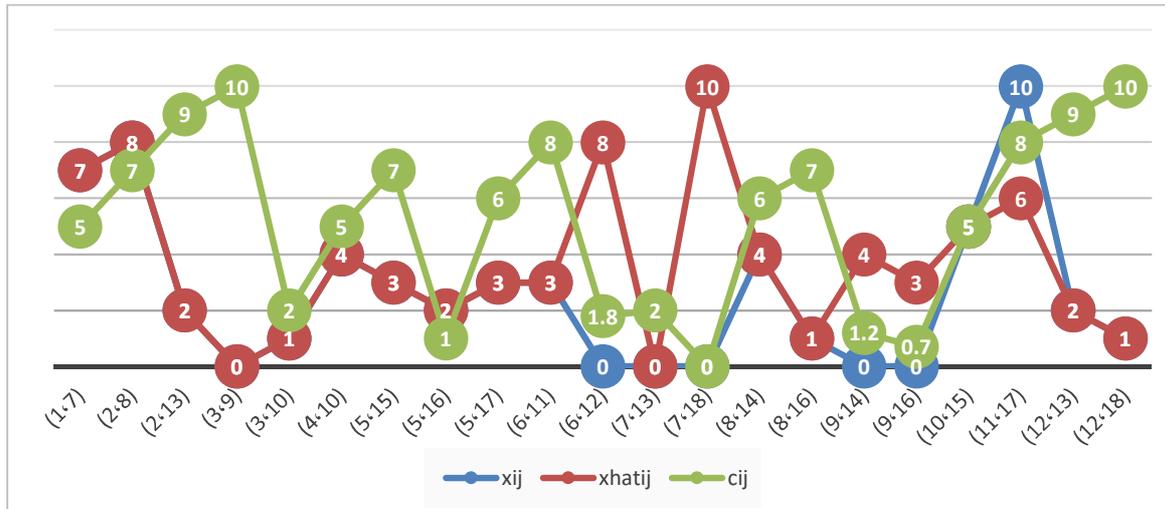
Table 8 reports the obtained results for CMFNIAI problem. Figures 4a, 4b and 4c show the flow rate per arc for the first scenario ( $x_{0ij}^1 = 0$ ), second scenario ( $x_{0ij}^2 = 5$ ) and third scenario ( $x_{0ij}^3 = 10$ ), respectively.



(4-a)



(4-b)



(4-c)

**Fig.4.** Results of CMFNIAI problem for different scenarios (a)  $s = 1$  (b)  $s = 2$  (c)  $s = 3$

As seen in **Fig. 4a**, the smuggler spend money and will be informed of the absence of police interdiction on the arc (3.10). Thus he/she can pass all possible flow through the arc. In other arcs, the smuggler trusts his/her initial estimation and does not spend any money for more accurate information on interdiction status. According to budget limits, the police setup the checkpoints in some arcs except the arcs (3.9), (6.12), (7.13), (7.18), (9.14) and (9.16) where they have the impression that the smuggler has overestimated the number of checkpoints and thereby does not pass the flows. If the police impression about the smuggler's reaction is not accurate, or this confidential information is leaked, then the smuggler is able to cross with maximum flow.

As seen in **Fig. 4b**, the smuggler has not spent money to buy the police information (from the black market) on the number of inspection stops and relies on his/her initial estimation in the arcs (1.7), (2.8) and (11.17). Since, the number of police stops is more than smuggler's estimation, the police can successfully interdict the flow.

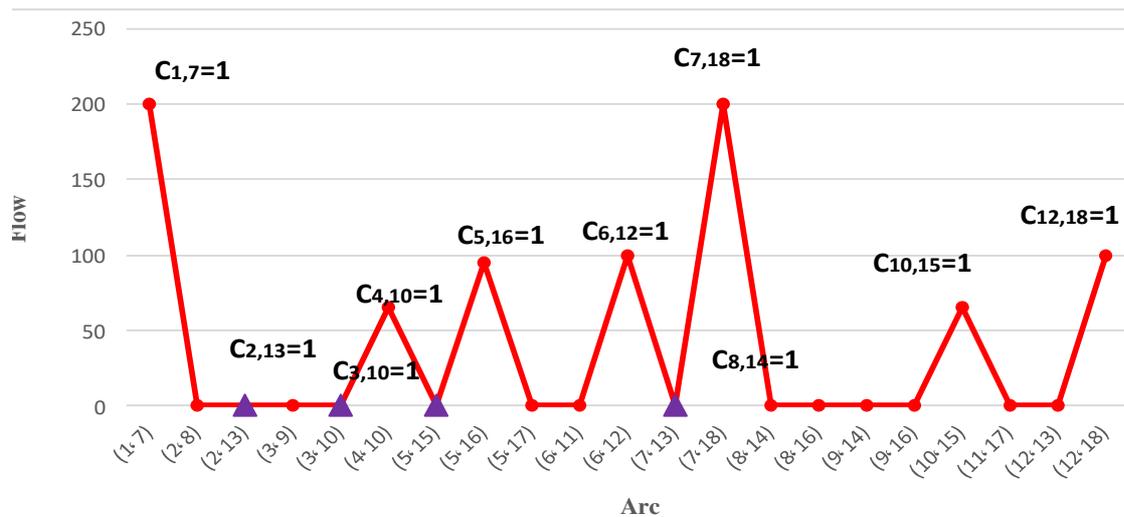
In arcs (3.9), (6.12), (7.13), (7.18) and (9.16), the police have not set up any checkpoints (probably because of budget limitation). The smuggler did not pay dirty money for the interdiction information (probably because of budget limitation), and relies on his initial overestimated impression. So, he/she cross these arcs with smuggling goods, conservatively (the smuggling goods flow is therefore reduced). In fact, if he had already paid for more accurate information, and was aware of non-interdiction on that arcs, he could pass more smuggling goods. Hence, police could successfully reduce the flow with no checkpoints.

The smuggler has spent money to get accurate information on the number of checkpoints in the arcs (2.13), (3.10), (5.15), (5.16), (5.17), (6.11), (8.14), (8.16), (12.12) and (12.18). Therefore, he/she would pass the smuggling goods and avoid the high-risk arcs, accordingly.

As seen in **Fig. 4c**, the smuggler does not pay for the accurate information on the number of stops in the arc (7.18), So, he trusts his initial estimation, and does not cross the arc, while the police have not established any checkpoint there. Finally, in the arcs (6.12), (9.14) and (9.16), the smuggler spent some money and found that the number of checkpoints is fewer than his/her initial estimate. Anyway, the police checkpoints are still there, so the smuggler try to pass the arcs conservatively (the smuggling goods flow is successfully reduced), and probably utilize other arcs alternatively.

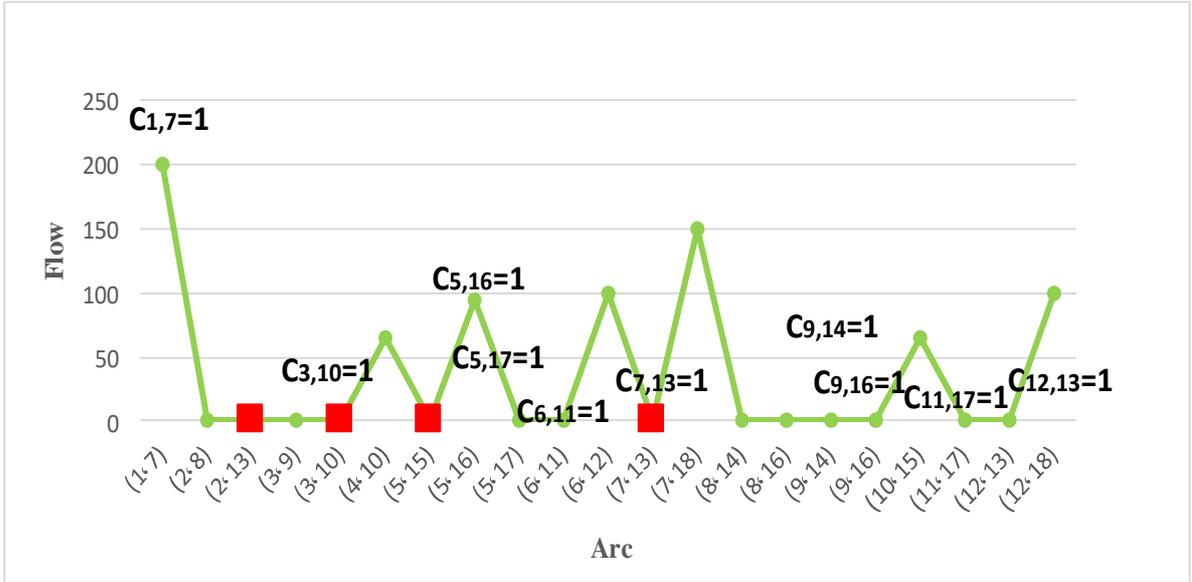
### 5.3.2. Asymmetric Information on DMFNIAI problem

The results of the DMFNIAI problem are extracted from Table 6 and depicted in Figures 5 and 6 to show the flow rate per arc for the first and second scenarios, respectively. The arcs in which the smuggler has payed to estimate the number of checkpoints are shown by the circle and that have been interdicted by the police are shown by the triangle and rectangle.



**Fig.5.** Results of DMFNIAI problem under the first scenario ( $x_{0ij}^1 = 1$ )

As Figure 5 shows, under scenario 1, the smugglers did not rely on his initial estimate in the arcs (1.7), (4.10), (5.16), (6.12), (7.18), (10.15) and (12.18), so he paid to be informed about the police actions, so he passes the flow, unfortunately. In this case, the police thought that the smugglers would act according to his initial estimate and therefore failed to interdict the smuggling. Conversely, in the arcs (2.8), (3.9), (5.17), (6.11), (8.16), (9.14), (9.16) and (11.17), the police have not set up any checkpoints but the smuggler erroneously realises on his initial estimate and avoid the arcs which are in fact not interdicted by the police. So, the police successfully interdict these arcs with nothing.



**Fig.6.** Results of DMFNIAI problem under the second scenario ( $x_{0ij}^2 = 0$ )

Similarly, **Figure 6** shows the flow rate per arc under the second scenario. The smugglers have spent money to nose around the arcs (1.7), (3.10), (5.16) and (7.13), They are informed about the police interdict in the arcs (3.10) and (7.13), and avoid them, and pass the desired flow through the arcs (1.7) and (5.16), because of the lack of police interdiction.

It is noteworthy that the scenarios under which the test problems are solved, are defined randomly, but in real world applications, the scenarios are definitely designed by the law enforcement subject-matter expert based on the historical data and/or their unique insights.

#### 5.4. Sensitivity Analysis on the Police Budget ( $B_p$ ) vs. smuggler budget ( $B_s$ )

We use the same test problem (Table 7) to investigate the effect of budget available to the police ( $B_p$ ) and smugglers ( $B_s$ ) on their actions in CMFNIAI model. We set their initial budget as 500 and change it by a multiplier  $\alpha$  ranges from 0.25 to 2 while the other parameters are supposed to be constant. After solving the CMFNIAI model optimally, the obtained results are depicted in Figure 7.

According to **Figure 7**, as expected, when the police budget increase, the objective function value of the CMFNIAI problem is reduced (Fig. 7-a), meaning that the smuggling flow is interdicted since the police can set up more checkpoints. On the other hand, when we encounter with bigger cartels (Fig. 7-b), they can spend more money to access the police checkpoints and other confidential information, which makes it more difficult for police to interdict the arcs successfully, and they should thereby make use of deception operations.



**Fig.7.** Sensitivity of CMFNIAI model to the police and smuggler budget

#### 5.4. Examining the efficiency of DM-PHA in more instances

In order to investigate the performance of the DM-PHA algorithm, 6 more test problems with small to large scales are solved, and the results are reported in Table 10. The first column shows the problem number and the next five columns show the dimensions of the problem. The number of iteration and computational time are reported in two last columns. It should be mentioned that the iterations of the PHA (outer loop of the algorithm) is reported here. As seen in Table 10, except problem number 6, the DM-PHA algorithm can solve the proposed two-stage bi-level model in reasonable time and with limited number of iterations.

**Table 10.** Performance analysis of the DM-PHA algorithm

#P	Nodes			Commodities	Scenarios	Objective Value	#Iteration	Solution time (se)
	Origins	Middle nodes	Destinations					
1	3	5	3	3	2	1,102	9	0:01:02:17
2	5	17	4	5	3	3,872	23	0:06:22:23
3	8	25	4	8	4	9,250	57	0:58:29:06
4	10	32	10	10	7	26,022	96	2:11:34:18
5	13	38	12	11	10	48,863	113	2:45:41:06
6	15	50	15	15	20	64,222*	207	5:00:00:00

\* CPLEX cannot reach the optimal solution and the best feasible solution after five hours is reported.

## 6. Conclusions, limitations and recommendations

This study presents two bi-level two-stage mixed-integer models in order to interdict the smuggling goods in a network under continuous and discrete interdiction setting. Budget limits, multiple origin and

destination, multiple commodities, uncertainty environment and asymmetric information of the players are considered simultaneously. The asymmetric information of the police toward smuggler and that of the smuggler toward police are formulated by a scenario-based approach and a linear function, respectively. The concepts of the risk-averse and risk-seeker smugglers are both taken into account. To solve the discrete and continuous interdiction models two innovative algorithms i.e. DM-PHA and augmented reformulation are developed. The augmented reformulation has been compared with KKT conditions and the results showed that the augmented reformulation is more efficient and needs less computational time. A series of sensitivity analyses are also performed and the applicability and efficiency of the proposed models as well as the solution methods are demonstrated. The results showed that the proposed models can provide appropriate solutions to successfully interdict the flow of smuggling goods. By the scenario-based approach the police are able to perform the deception operation, meaning that they interdict the flow of an arc with no many checkpoints.

Despite the added value of the proposed models in the context of information asymmetry, there are limitations that need to be acknowledged. Reviewing the literature reveals that only few papers have considered information asymmetry in the network interdiction problems, in which the type of asymmetry is different from this research, they have also assumed different objective functions (e.g. shortest path, minimum cost). It therefore limits the researchers to precisely compare the results. Another shortcoming of this study is related to data collection. Due to the secretive nature and lack of verifiable data on illicit trade, it is difficult to compute with absolute precision the market size of smuggling. Additionally, it is impossible to deal with the world of trafficking, physically monitor, control, and secure borders through manpower alone. This research is more applicable for the implementation of anti-smuggling and border patrol agencies assuming that they are equipped with advanced technologies, such as unmanned aerial vehicles (UAV), embedded sensor and actuator solutions in transport assets, x-ray technologies, and even robotics (Basu, 2014).

Possible future research paths on interdiction problem may be pursued in: (i) considering different interdiction priorities for the different commodities based on the interdictor's preferences, (ii) proposing other functions with which the information asymmetry of the smuggler toward police is formulated, for instance staircase, piecewise or nonlinear functions, and (iii) applying the proposed information asymmetry in other variants of network interdiction problem such as shortest paths and minimum cost.

## **Appendix A**

According to Mirzapour Al-e-Hashem et al. (2013), and by introducing a new positive auxiliary variable  $U_{ij}^s$  defined in (A.1)

$$\text{Max}\{x_{ij}, x_{ij} \cdot c_{ij}^s\} \rightarrow U_{ij}^s \quad \forall (i, j) \in A, \forall s \in S \quad (\text{A.1})$$

, DMFNIAI model can be linearized as follows:

$$\text{Minimize } E[f^s(x)] \quad (\text{A.2})$$

s.t.

$$\sum_{ij} h_{ij} x_{ij} \leq Bp \quad (\text{A.3})$$

$$f^s(x) = \text{Maximize } E\left[\sum_{i \in N, j \in D, k \in K, s \in S} y_{ijk}^s\right] \quad (\text{A.4})$$

s.t.

$$\sum_i y_{ijk}^s - \sum_i y_{jik}^s = 0 \quad \forall j \in V, \forall k \in K, \forall s \in S \quad (\text{A.5})$$

$$\sum_{k \in K} y_{ijk}^s \leq u_{ij}(1 - x_{ij}) \quad \forall (i, j) \in A, \forall s \in S \quad (\text{A.6})$$

$$\sum_{k \in K} y_{ijk}^s \leq u_{ij}(1 - (U_{ij}^s + (1 - c_{ij}^s)x_{0ij}^s)) \quad \forall (i, j) \in A, \forall s \in S \quad (\text{A.7})$$

$$\sum_{ij} c_{ij}^s \cdot cm_{ij} \leq Bs \quad \forall s \in S \quad (\text{A.8})$$

$$U_{ij}^s \leq c_{ij}^s \quad \forall (i, j) \in A, \forall s \in S \quad (\text{A.9})$$

$$U_{ij}^s \leq x_{ij} \quad \forall (i, j) \in A, \forall s \in S \quad (\text{A.10})$$

$$x_{ij} + c_{ij}^s - 1 \leq U_{ij}^s \quad \forall (i, j) \in A, \forall s \in S \quad (\text{A.11})$$

$$x_{ij}, c_{ij}^s, U_{ij}^s \in \{0, 1\} \quad \forall (i, j) \in A, \forall s \in S \quad (\text{A.12})$$

$$y_{ijk}^s \geq 0 \quad \forall (i, j) \in A, \forall k \in K, \forall s \in S \quad (\text{A.13})$$

## Appendix B

First, the discrete variable  $x_{ij}$  is binarized by the help of an auxiliary binary variable  $t_{ijk}$  (see Eq. B.1)

$$x_{ij} = \sum_{l=0}^{n-1} 2^l t_{ijk} + (U_{x_{ij}} - 2^n + 1)t_{ijn} \quad (\text{B.1})$$

So, the bilinear term of  $x_{ij} \cdot c_{ij}^s$  can be rewritten as:

$$x_{ij} \cdot c_{ij}^s = \sum_{l=0}^{n-1} 2^l t_{ijk} \cdot c_{ij}^s + (U_{x_{ij}} - 2^n + 1)t_{ijn} \cdot c_{ij}^s \quad (\text{B.2})$$

Where,  $t_{ijk} \cdot c_{ij}^s$ , itself is a nonlinear term. Hence, as the second step, we linearize this term by introducing a new positive auxiliary variable  $z_{ijk}$ :

$$t_{ijk} \cdot c_{ij}^s \rightarrow z_{ijk} \quad (\text{B.3})$$

$$x_{ij} \cdot c_{ij}^s \rightarrow \sum_{l=0}^{n-1} 2^l z_{ijk}^s + (U_{X_{ij}} - 2^n + 1) z_{ijk}^s \quad (\text{B.4})$$

Therefore, by adding the following constraints:

$$c_{ij}^s - (1 - t_{ijk}) \cdot M \leq z_{ijk}^s \quad \forall (i, j) \in A, \forall k \in K, \forall s \in S \quad (\text{B.5})$$

$$z_{ijk}^s \leq c_{ij}^s \quad \forall (i, j) \in A, \forall k \in K, \forall s \in S \quad (\text{B.6})$$

$$z_{ijk}^s \leq M \cdot t_{ijk} \quad \forall (i, j) \in A, \forall k \in K, \forall s \in S \quad (\text{B.7})$$

$$t_{ijk} \in \{0,1\} \quad \forall (i, j) \in A, \forall k \in K \quad (\text{B.8})$$

$$z_{ijk}^s \geq 0 \quad \forall (i, j) \in A, \forall k \in K, \forall s \in S \quad (\text{B.9})$$

The equivalent linearized CMFNIAI model could be written as follows:

$$\text{Minimize } f^s(x) \quad (\text{B.10})$$

s.t.

$$\sum_{i,j} h_{ij} \sum_{l=0}^{n-1} 2^l t_{ijk} + (U_{X_{ij}} - 2^n + 1) t_{ijn} \leq Bp \quad (\text{B.11})$$

$$\sum_{k \in K} y_{ijk}^s \leq u_{ij} \left( 1 - \left( \frac{\sum_{l=0}^{n-1} 2^l t_{ijk} + (U_{X_{ij}} - 2^n + 1) t_{ijn}}{U_{X_{ij}}} \right) \right) \quad \forall (i, j) \in A, \forall s \in S \quad (\text{B.12})$$

$$\sum_{k \in K} y_{ijk}^s \leq u_{ij} \left( 1 - \left( \frac{x_{0ij}^s}{U_{X_{ij}}} \right) \right) \quad \forall (i, j) \in A, \forall s \in S \quad (\text{B.13})$$

$$+ \frac{\sum_{l=0}^{n-1} 2^l z_{ijk}^s + (U_{X_{ij}} - 2^n + 1) z_{ijk}^s}{U_{X_{ij}} \cdot cm_{ij}} - \frac{x_{0ij}^s}{U_{X_{ij}} \cdot cm_{ij}} c_{ij}^s))$$

, and Constraints 13-14, 16-20, 29-33

### Appendix C (Relaxed Smuggler Problem)

In order to build the  $\overline{\text{DMFNIAI}}$  model (step 2) we first need to have the dual model of the follower problem (Smuggler problem). The smuggler problem (inner level) with continuous variables is as follows:

$$Max \left( \sum_{i \in N, j \in D, k} y_{ijk} - M \sum_{i \in N, j \in D, k} y_{ijk} \cdot x_{ij} - M \sum_{i \in N, j \in D, k} y_{ijk} \cdot x_{ij} \cdot c_{ij} \right) \quad (C.1)$$

$$- M \sum_{i \in N, j \in D, k} y_{ijk} \cdot x_{0ij} + M \sum_{i \in N, j \in D, k} y_{ijk} \cdot c_{ij} \cdot x_{0ij}$$

s.t.

$$\sum_{k \in K} y_{ijk}^s \leq u_{ij} \quad \forall (i, j) \in A \quad (C.2)$$

$$y_{ijk}, c_{ij} \geq 0 \quad \forall (i, j) \in A, \forall k \in K \quad (C.3)$$

, and Constraints 5,7

#### Appendix D (Dual model of lower level problem)

$$Min u_{ij} \cdot \Omega_{ij} + Bs. \theta \quad (D.1)$$

s.t.

$$\begin{aligned} \pi_{ik} - \pi_{jk} + \Omega_{ij} \geq 1 - M \sum_{i, j \in V, k} x_{ij} - M \sum_{i, j \in V, k} x_{ij} \cdot c_{ij} \\ - M \sum_{i, j \in V, k, s} x_{0ij} + M \sum_{i, j \in V, k} c_{ij} \cdot x_{0ij} \end{aligned} \quad \forall (i, j) \in A, \forall k \in K \quad (D.2)$$

$$\begin{aligned} \pi_{ik} + \Omega_{ij} \geq 1 - M \sum_{i \in V, j \in D, k, s} x_{ij} - M \sum_{i \in V, j \in D, k, s} x_{ij} \cdot c_{ij} \\ - M \sum_{i \in V, j \in D, k, s} x_{0ij} + \sum_{i \in V, j \in D, k} c_{ij} \cdot x_{0ij} \end{aligned} \quad \forall (i, j) \in A, \forall k \in K \quad (D.3)$$

$$\begin{aligned} -\pi_{jk} + \Omega_{ij} \geq 1 - M \sum_{i \in O, j \in V, k} x_{ij} - M \sum_{i \in O, j \in V, k} x_{ij} \cdot c_{ij} \\ - M \sum_{i \in O, j \in V, k} x_{0ij} + M \sum_{i \in O, j \in V, k} c_{ij} \cdot x_{0ij} \end{aligned} \quad \forall (i, j) \in A, \forall k \in K \quad (D.4)$$

$$\sum_{i, j} cm_{ij} \cdot \theta \geq M \sum_{i \in N, j \in D, k} y_{ijk} \cdot x_{0ij} - M \sum_{i \in N, j \in D, k} y_{ijk} \cdot x_{ij} \quad (D.5)$$

$$\Omega_{ij}, \theta, c_{ij} \geq 0 \quad \forall (i, j) \in A \quad (D.6)$$

$$x_{ij} \in \{0, 1\} \quad \forall (i, j) \in A \quad (D.7)$$

$$\pi_{jk}: urs \quad \forall j \in V, \forall k \in K \quad (D.8)$$

## Appendix E

The dual of the lower level problem for CMFNIAI is as follows:

$$\begin{aligned} \text{Min } u_{ij} \left( 1 - \left( \frac{x_{ij}}{UX_{ij}} \right) \right) \cdot \Omega 1_{ij}^s + u_{ij} \left( 1 - \left( \frac{x_{0ij}^s}{UX_{ij}} \right) \right) \Omega 2_{ij}^s \\ + Bs. \theta 1^s + cm_{ij} \cdot \theta 2_{ij} \end{aligned} \quad (\text{E.1})$$

s.t.

$$\pi_{jk}^s - \pi_{ik}^s + \Omega 1_{ij}^s + \Omega 2_{ij}^s \geq 1 \quad \forall (i, j) \in A, \forall k \in K, \forall s \in S \quad (\text{E.2})$$

$$-\pi_{jk}^s + \Omega 1_{ij}^s + \Omega 2_{ij}^s \geq 1 \quad \forall i \in O, \forall j \in V, \forall k \in K, \forall s \in S \quad (\text{E.3})$$

$$\pi_{ik}^s + \Omega 1_{ij}^s + \Omega 2_{ij}^s \geq 1 \quad \forall j \in D, \forall i \in V, \forall k \in K, \forall s \in S \quad (\text{E.4})$$

$$\frac{x_{ij} \cdot u_{ij}}{cm_{ij} \cdot U_{X_{ij}}} \cdot \Omega 2_{ij}^s - \frac{x_{0ij}^s \cdot u_{ij}}{cm_{ij} \cdot U_{X_{ij}}} \cdot \Omega 2_{ij}^s + \theta 1 + \theta 2_{ij} \geq 0 \quad \forall i, j \in N, \forall k \in K, \forall s \in S \quad (\text{E.5})$$

$$\Omega 1_{ij}^s, \Omega 2_{ij}^s, \theta 1, \theta 2_{ij} \geq 0 \quad (\text{E.6})$$

$$\pi_{jk}: \text{urs} \quad (\text{E.7})$$

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